

Ce que penser veut dire ?
**Cavallès and the problem of the link
between philosophy and mathematics**

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I'm not trying to define mathematics, but, by means of mathematics, to find out what it means to know, to think; this is basically, very modestly borrowed, the problem of Kant.^()*

^(*)Jean CAVAILLES, in his lecture at the Société française de philosophie (session of February 4, 1939)

It is not our intention in this article to review Cavallès's famous "Spinozism", nor his more hidden (and more recently analyzed) "Hegelianism".⁽¹⁾ Spinoza will reappear here and there, however, insofar as his philosophy seems to us to shed light on the Cavallèsian conception of "thought", and to distinguish it from other conceptions. Our aim in this study is to evaluate the quotation in exergue, and to reflect through it on the problem of the relationship between philosophy and mathematics in Cavallès' work. In particular, the question will be how to interpret the phrase "by means of mathematics" in this quotation. Does this mean that, for Cavallès, mathematics is the proper framework for the expression of thought or, as we shall argue here, that it constitutes a privileged, more "revealing" (but not exclusive) expression of thought? As the reference to Kant reminds us, Cavallès' thinking is part of a project to criticize knowledge based on mathematics. This reference undoubtedly owes much to the Brunschvicgian reading of the Kantian project — a reading that made mathematics the linchpin of the *Critique de la raison pure*, and thus saw this work first and foremost as a work of "mathematical philosophy". But this is more a way of setting out the terms of the problem than indicating its resolution. For Cavallès, as for Brunschvicg, philosophy and mathematics seem to go hand in hand, and thought accompanies, and even blends into, the progress of science. But how do these notions relate to each other? Do mathematics and philosophy merge in thought? Is mathematics more properly identified with the latter? But if thought is revealed, and perhaps elaborated, through the mathematical process, what place then can be assigned to philosophical investigation (can it even be considered as an activity of "thought"?)?

To address these questions, we'll be drawing not only on Cavallès' doctoral work (whose quotation at the beginning of this

⁽¹⁾ Readers interested in these questions may refer to Hourya Sinaceur's works, notably *Jean Cavallès. Philosophie mathématique* (PUF, 1994) and *Cavallès* (Les Belles Lettres, 2013); to Elisabeth Schwartz's study "Le "testament philosophique" de Jean Cavallès: vers une Logique de la création?" (PUF, "Revue de métaphysique et de morale", 2020, pp. 165-198); and to the "choral" work *Pour Cavallès* (Pont 9, 2021), by Christian Houzel, Didier Nordon, Xavier-Francaire Renou, Henri Roudier and Jean-Jacques Szczeciniarz, several sections of which are explicitly devoted to a characterization and analysis of Cavallèsian influences (notably the "Second Movement" of Book I).

paper is an extract from the review),⁽²⁾ but also, and even mainly, in the work referred to by the "condemned man from Arras" as his "philosophical testament"⁽³⁾ and posthumously entitled *Sur la logique et la théorie de la science*. The aim will thus be to take into account the pursuit of research whose exploratory and "insufficient" character was humbly emphasized by Cavailles at the end of his dissertations,⁽⁴⁾ and to attempt to identify an overall *orientation* of response. We say "orientation" because Cavaillesian exegesis poses three intertwined difficulties, making the reconstruction of a complete and coherent perspective illusory. The first difficulty is, of course, the unfinished nature of a work that was still in gestation at the time of its author's very premature death. The second difficulty, which is not fully explain by the first, lies in Cavailles' own syncretic and sometimes allusive style. Finally, the third difficulty lies in the fact that we are dealing with a work that is clearly *evolutive*. To borrow the famous formula of *Sur la logique* characterizing scientific progress, we could say that Cavailles' thinking itself proceeds by "deepening and erasing" (an analogy that may well prove meaningful for our purpose).

§ — Problem position.

The double danger of linking philosophy to mathematics. As soon as we start reading Cavailles's first texts, the astonishment — essentially methodological — we may feel at his will to "stick" closely to the mathematical path, already plunges us to the heart of our problem. Indeed, we may have the feeling that he is not approaching the problems from a philosophical point of view. Yet it seems to us that this should be his point of view, given that the task assigned to these texts is not directly to provide a mathematical solution to the foundational problems at the heart of the subject. The Foundational

⁽²⁾This review, in which Cavailles and Lautman set out and discussed the results of their respective dissertations, was published under the title "La pensée mathématique": cf. pp. 593 to 630 in Jean Cavailles, *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994 (p. 625 for the quotation at the beginning of this paper).

⁽³⁾It was when he handed the manuscript to his sister that he referred to it in these words. Cf. the biographical work: G. Ferrières, *Jean Cavailles — un philosophe dans la guerre*, éd. du Félin, coll. Résistance Liberté-Mémoire, Paris, 2003.

⁽⁴⁾"La pensée mathématique" in Jean Cavailles, *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 604, 625, 627.

Crisis is a mathematical crisis, Cavailles seems to tell us at the start of his main dissertation; and yet, he ends this dissertation by emphasizing that it is essentially a philosophical problem.

Bernays denied in 1934 — shortly after the disappointment of formalist ambitions — that there was a crisis in mathematics: "in truth, the mathematical sciences are growing in full security and harmony". A philosophical crisis only because extrinsic demands were made (...). Hence, no doubt, also a good deal of exaggeration in the difficulties of set theory: really there are, it seems, only those arising from the mixture of philosophical speculation and mathematical reasoning and those, normal, caused by technical inadequacies.⁽⁵⁾

The crisis of foundations thus confronts us with the problem of philosophical interference in mathematics.

Generally speaking, Cavailles sees a double danger in linking philosophy to mathematics:

1. To insert itself directly into their own development, believing that mathematical technique can, by itself, answer the philosophical questions raised about it.
2. To integrate them into a more global interpretation that assigns to mathematics a function and nature that are not themselves mathematical (and yet generally lead to the establishment of norms of validity governing technique).

The first danger in fact characterizes the erasure of the philosophical in the mathematical, which, if it fails, leads to the opposite situation, i.e. the second danger, in which it is philosophy that this time legislates on mathematics. In both cases, one field encroaches on the other, with mathematics alternately becoming the only true "philosophy" or a mere extension of it. The fundamental problem here is that mathematics and philosophy are both conceived as belonging to the same abstract field of knowledge; hence the permeability of the two approaches (each seeking to establish its pre-eminence over the other), linked to this presupposition of a possible congruence of their problems. Science and philosophy as (in this case) "theory of science"

⁽⁵⁾ *Méthode axiomatique et formalisme* in *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 189/190.

have the same "claim to validity and intelligibility";⁽⁶⁾ how, then, can we "situate" the two processes respectively?

The status of a "theory of science"? In Cavallès' posthumous work (which is also his "philosophical testament"), the analysis of Bolzano's doctrine occupies a special place.⁽⁷⁾ Indeed, it provides the opportunity for one of the most explicit expositions of his own theory of science. And, in particular, it was on the basis of his research into Bolzano's thought that Cavallès identified the decisive problem of the status of a theory of science, and provided some elements for resolving it.

The difficulty immediately arises, not only in justifying and specifying these characteristics [the highlighted characteristics of science], but also in situating the discipline that posits them. The doctrine of science also has a claim to validity and intelligibility; it would be the science of science, and therefore part of science itself. Its statements, then, are not constitutive of a particular development, but appear immediately in a self-illumination of the scientific movement, yet are distinguished from it by their permanent emergence. This is the role of the structure. By defining a structure of science that is only a manifestation to it of what it is, we clarify and justify the preceding characteristics, not by an explicitness that would have its own place and would, in turn, be an object of reflection, but by a revelation that is not distinct from the revealed, present in its movement, principle of its necessity. The structure speaks about itself.⁽⁸⁾

⁽⁶⁾ *Sur la logique et la théorie de la science*, Vrin, Paris, 1997 (second edition), p. 39. The following quotations from this work refer to the same edition.

⁽⁷⁾ Cf. "Bolzano considers — and almost succeeds in solving — the same problems of the mathematical legitimacy (...). For the first time, perhaps, science is no longer seen as a mere intermediary between the human mind and the being-in-itself, depending as much on one as on the other, and having no reality of its own, but as a *sui generis* object, original in its essence, autonomous in its movement" (*Sur la logique et la théorie de la science*, Vrin, Paris, 1997 (second edition), p. 36). The link with the autonomy of mathematical experience, as defended by Cavallès (and on which we will dwell in this study), is clear here. This makes the difficulties faced by the Bolzanian doctrine all the more decisive — since they constitute the problems to be solved by anyone wishing to establish an autonomous conception of science.

⁽⁸⁾ *Ibid.*, p. 39.

It is the very possibility and legitimacy of the work he does that Cavaillès considers here. What is really at stake is the safeguarding of a discourse on mathematics (since it reveals its "structure") which, while remaining within the realm of science, would be capable of bringing Cavaillès' "Kantian project" to fruition: finding out what it means to "know", to "think". The legitimacy in question would therefore correlatively establish that of a philosophical approach to mathematics, at once immersed in their movement and yet distinct and revealing from it. As we have seen, this legitimacy rests on "a self-illumination of the scientific movement", the very "principle of the necessity" that characterizes it. But how is this to be understood? Does it mean that only this movement is susceptible to objectivity, and therefore that its structure can only be a reaffirmation of it (first danger)? But then, theorizing no longer has any real meaning, and all we can do is accompany the movement without questioning it further. Hence the second danger, which is inherent in the ambition to theorize: characterizing scientific development presupposes a certain "step back", a certain detachment; the risk then being to lose the objectivity, the intelligibility immanent in this development (which is compensated for by the establishment of arbitrary norms).

Cavaillès warns against both sides of this alternative, which means that his solution must be able to navigate between the two pitfalls distinguished here: a theorization entirely absorbed in mathematics, or doomed to the emptiness of normative arbitrariness. In other words, it is a question of re-establishing a boundary blurred by the crisis, while preserving a domain of philosophical expressivity (and therefore, correlatively, the possibility of a theorization of science) that is not cut off from scientific objectivity and intelligibility. In the case of a doctrine of science, whose aim is precisely to be able to characterize this objectivity, "its statements are not constitutive of a particular development, but appear immediately in a self-illumination of the scientific movement, yet are distinguished from it by their permanent emergence." In this way, two levels are both distinguished and conciliated: that of pure mathematical effectivity, and that of theoretical and philosophical discourse on this effectivity, which in fact consists in allowing it to self-reveal itself by tracing, not its historical development, but the ever-renewed necessity of the sequences that preside over this development. Thus, the philosophical "step back" demanded by a

doctrine of science is not a step back from mathematical objectivity, but a self-manifestation of the "principle of its necessity": this means that the distancing between the two processes must not cancel out their community of necessity, otherwise the repercussion of mathematical objectivity on the theoretical level of a doctrine of science will be invalidated. This study will seek to clarify this simultaneous safeguard.

§ 1. — The identification of "mathematical thought" with "thought" (first danger).

First of all, isn't this desire for absolute respect for mathematical immanence, combined with an investigation into the nature of thought, a risk of exposure to the first danger? Doesn't this mean that mathematical thought is completely identified with thought, and that there is no true thought outside mathematics? In which case, mathematics would be the only true philosophy, or, more precisely, there would be no philosophy at all, since it would be subject to mathematical immanence, the only authentic expression of thought.

As problematic as it may be, such an interpretation of the Cavallèsian perspective is nonetheless attractive, and solidly referenced. In fact, it is very similar to that advocated by Pierre Cassou-Noguès in his article "Consciousness and reflexivity in Cavallès' mathematical philosophy":

In his lecture to the Société française de Philosophie, Cavallès seems to reduce all thought to mathematical thought: "There is no effectively thought representation (distinct from pure experience) that is not a mathematical system insofar as it is thought — that is, regulated organization of the sensible [...]".⁽⁹⁾ A mathematical system is, for Cavallès, a system of signs and rules of use, whereas pure experience is a field of "lived impressions, rigorously untranslatable, rigorously unusable by means of a rule."⁽¹⁰⁾ But thought is expressed in signs and, insofar as it cannot be reduced to pure experience,

⁽⁹⁾Jean Cavallès, *Oeuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 594 (in "La pensée mathématique").

⁽¹⁰⁾*Ibid.*, p. 625.

it is expressed in a system of signs and rules, i.e. as a mathematical system. Mathematics is not characterized by a few fixed demonstrative forms. Mathematics is characterized by the existence of rules that govern the use of signs and make operations communicable and verifiable. In this sense, all authentic thought is mathematical. Philosophy, in particular, is mathematical. Philosophy is a system of signs, the words of language, and of rules, the usual rules of grammar and the specific rules that determine the meaning given to words. The difficulty is that, if the philosophical text is subject to rules as strict as those of mathematical demonstration, the philosopher cannot show the path that leads from the usual language or the language of the philosophers who preceded him to his own language. Indeed, to move from one language to another, he would have to modify the rules little by little, as he progressed, which would deprive philosophy of its mathematical character. The philosophical text develops a constructed language without being able to construct it. The philosopher proceeds by aligning the formulas of his language without being able to deliver the keys. (...).⁽¹¹⁾

Such an analysis of thought, and the associated critique of philosophy, brings the Cavallèsian perspective remarkably close to that of Wittgenstein.⁽¹²⁾ There's nothing incongruous about this: many commentators have stressed the points of convergence between the two authors. In particular, their characterizations of mathematical development often coincide.⁽¹³⁾ Particularly salient in their work is the problem of the relationship between the philosophical and

⁽¹¹⁾ Pierre Cassou-Noguès, "Consciousness and reflexivity in the mathematical philosophy of Cavallès", *Methodos* [Online], 1 | 2001, online April 05, 2004, alinea 33.

⁽¹²⁾ It should be emphasized that this comparison is not made by Pierre Cassou-Noguès, nor does it fit in with the rest of his paper, whose innovative aim is to show that, in Cavallès, "consciousness is defined and constituted" on the basis of "the reflexivity of mathematical becoming".

⁽¹³⁾ It should be noted that Cavallès was probably the first French philosopher to grasp the importance of Wittgenstein's work (limited at the time to the *Tractatus logico-philosophicus* and the *Remarks on Logical Form*) and to provide an in-depth analysis of it. To appreciate the importance Cavallès attached to the latter, we need only refer to his review of the Prague International Congress of Philosophy (held in 1934) and, throughout his work, note the significance of certain Wittgensteinian theses for the critique of logicism as well as Husserlian philosophy. It is important, however, to distinguish clearly between "two" Wittgensteins: the one that Cavallès actually addresses, and the one that we can

the mathematical, and, more precisely, of the legitimacy (and, in this case, the modalities) or illegitimacy of the insertion of one into the other. For both authors, this problem, already mentioned in the introduction, is the mainspring of the Foundational Crisis. Insofar as this polemic can be explained by the (parasitic?) interference of philosophical discourse in mathematics, is there really any place left for philosophical discourse within mathematics? And if so, how can it be determined and established? From his earliest works, and through his very method, Cavaillès seeks to capture the mathematical movement from within: let's not interfere in the process of mathematical development, he seems to be telling us. This approach, which is also found in Wittgenstein's work, implies a number of analyses of the relationship between philosophy and mathematics, which we'd like to spell out more clearly here. In particular, and to return to the problem of "thought", should we subscribe to the identification of "thought" with "mathematical thought" alone (dooming philosophy, if it wants to participate in thought, to be resorbed in mathematics)?

"Tentacular" mathematics. We don't believe that this is the position defended by Cavaillès. In fact, when he refers to Brouwer's intuitionism, he warns against the totalizing dimension of the mathematical concept as organizing power par excellence -which shows that he is well aware of this problem, and believes that his own theorization should help to avoid it.

But then the second question [for the intuitionist doctrine] arises: how to distinguish this [the mathematical movement] in the general march of science or even culture? "Mathematics, says Brouwer, is the ordering of the world, the rational thinking of the world" — without the terms "world", "rationality" being further subjected to criticism. In particular, the relationship with physics remains vague. If any science is determined as a type by

only guess at, but which we are now better able to analyze, thanks to later works that the "condemned man from Arras" never had the chance to consult. It would therefore be pointless to look for traces of this "second" Wittgenstein in Cavaillès. But it is nonetheless interesting to evoke him in a comparative perspective, as his philosophy of mathematics fits in perfectly with Cavaillès' formula: "to understand is to catch the gesture, and to be able to continue" (*Méthode axiomatique et formalisme*, p. 186).

the Brouwerian rule, it's because here again mathematics serves as organon, *but to such an extent that it absorbs the rest.*⁽¹⁴⁾

Cavaillès is referring here to Brouwer's lecture *Mathematik, Wissenschaft und Sprache*, whose general thesis expresses, in very Schopenhaurian terms, that everything that has to do with "order" or "organization" is a mathematical expression of the will.⁽¹⁵⁾ The approach is indeed extremely general, and even characterizes a certain "tentacular" dimension of mathematics, since it properly constitutes thought in its movement, i.e. in the exercise of its demi-urgic function of ordering the world — to such an extent that it's hard to see how mathematical thought and thought could not fully coincide.

For Brouwer, this coincidence was underpinned by a vision of mathematics as the exact science par excellence, i.e. as something that can be freed from linguistic uncertainties by being placed in the realm of intellectual intuition. This leads Brouwer to develop a dynamic conception of mathematics, which is nothing other than a free succession of creative acts (in marked opposition to the fixity and permanence of a "Platonism"). Its extra-linguistic existence aside, we find here some of the major themes in Cavaillès and Wittgenstein's characterization of mathematics. Does this mean that they also follow Brouwer regarding the assimilation of mathematics to thought?

What is certain is that both criticize mathematical foundations, preferring instead an immanentist investigation of the gestures, acts, decisions, problems — in short, the practice — of the "militant mathematician".⁽¹⁶⁾ Both question the status and scope of logical investigations and efforts to characterize and structure the mathematical "living tissue", and call for a "less absolute" meaning to be given to such processes. Both ultimately urge us to respect the autonomous,

⁽¹⁴⁾ *Sur la logique et la théorie de la science*; p. 32.

⁽¹⁵⁾ Given in Vienna in 1928 (published the following year), this lecture, probably the most famous of Brouwer's, is said to be the reason for Wittgenstein's return to Cambridge (after eight years of "silence"), where he began to elaborate his "second" philosophy.

⁽¹⁶⁾ Cavaillès, *Remarques sur la formation de la théorie abstraite des ensembles*, in *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p.362.

endogenous becoming of mathematics,⁽¹⁷⁾ while granting this becoming a special status within human productions, and making it a fertile source of lessons for knowledge. As Wittgenstein states, questioning in his own way the legitimacy and meaning of the work he is doing: "What I have to do is something like: write the function of a king; — and in doing so, I must not make the mistake of explaining royal dignity from the king's utility; nor must I neglect utility or dignity."⁽¹⁸⁾ In other words, even if the external application of the mathematical rule is particularly significant and should not be neglected, it cannot do justice to the necessity, autonomy and normativity of this rule. Wittgenstein's approach, likethat of Cavailles, thus places him on a razor's edge: constantly oscillating between respect for mathematical immanence and the endangerment of this immanence by both the distance that an investigation on mathematics seems to demand, and the central role that mathematics assumes outside its own "game". These considerations, which combine autonomy and the "tentacular" dimension of mathematics, are precisely what lies at the heart of the problem raised by the Brouwerian conception evoked by Cavailles. However, it is through their respective ways of avoiding this problem (in what it implies about the relationship between philosophy and mathematics) that we believe the gap between Cavailles and Wittgenstein will widen significantly. Should we identify these autonomous mathematical creations, which bring order and organization to the sensible, with thought? It's worth noting that, even if it's difficult to consider him, strictly speaking, as a "transcendental" philosopher, Wittgenstein, like Cavailles, also

⁽¹⁷⁾ Thus, for example, in Wittgenstein: "[A]rithmetic constructions are autonomous, like geometric constructions, and thereby, so to speak, guarantee their applicability themselves" (*Remarques philosophiques*, §111, p. 127). Or again: "the application of calculus must take care of itself. And this is what is correct in 'formalism'" (*Remarques sur les fondements des mathématiques*, third part, §4, p. 139). With regard to this reference to formalism, one of the reasons for the apparent closeness between Cavailles and Wittgenstein is undoubtedly their shared sympathy for Hilbertian perspectives. Although it would require a specific study, Hilbert's axiomatic method clearly played a decisive role in the development of Wittgenstein's method of grammatical investigation. As for Cavailles, he endeavors to reinvest, beyond the failure of the foundational programs, some original Hilbertian approaches (notably his "theory of the sign") within the framework of what he calls a "modified formalism". In short, both Cavailles and Wittgenstein could be said to have embarked on the path of a certain unorthodox formalism — devoid, in particular, of foundational ambitions.

⁽¹⁸⁾ *Remarques sur les fondements des mathématiques*, Part Seven, §3, p. 289.

claims to be part of a Kantian project to criticize knowledge.⁽¹⁹⁾ But in Wittgenstein, this project is inseparable from a radical critique of metaphysical discourse, and from a reactive conception of philosophy, envisaged not as a theoretical activity of thought, but as an effort to clarify and level out an actual use that needs to be left "as it is". Wittgenstein's aim is to bring words back from their metaphysical use to their everyday use, and in this context, "finding out what it means to think" would signify clarifying, not the thought itself, but what we usually give the name 'thinking' to — what we "call" thinking. This is explicitly what he says when he discusses the logicist idea that logical laws express the essence of thought (they are the "laws of thought"):

Logical laws are certainly the expression of habits of thought, but they are also the expression of the habit of *thinking*. In other words, they can be said to show: how men think, and also *what men call "thinking"*.⁽²⁰⁾

It is beyond the scope of this study to analyze the polysemy of Wittgenstein's notion of "logic". What is important here is that the perspective he develops excludes any attempt to uncover a structure of science (the very idea of a "theory" or "doctrine" of science appears irrelevant in Wittgenstein's perspective). In this sense, if an investigation into "what it means to think" does indeed lead to the logico-mathematical field, and appears to be inseparable from it, then for Wittgenstein it's only a question of showing "how men think" and what they "call thinking" (the operative rules to which we attribute the status of "rules of thought"). Now, even if Cavallès also criticizes philosophy — particularly philosophies of the subject or foundational, reductionist philosophies — this is not at all in the same terms as Wittgenstein's, since it does not exclude a theoretical discourse on the general structure of mathematics. In Cavallès' perspective, therefore, there seems to be room for a conception of philosophy as an activity of thought — as confirmed by his above criticism of the Brouwerian assimilation of mathematics to the "rational thinking of the world"

⁽¹⁹⁾ Thus, for example, in the *Investigations* (§ 90): "We have the impression that we should pierce through phenomena: our search, however, is not directed at phenomena, but, one might say, at the 'possibilities' of phenomena." Or again, in *Remarques mêlées* (T.E.R., 1984, p. 22): "The limit of language shows itself in the impossibility of describing the fact that corresponds to a proposition (which is its translation) without, precisely, repeating the proposition. (We are dealing here with the Kantian solution to philosophy)."

⁽²⁰⁾ *Remarques sur les fondements des mathématiques*, Part I, § 131.

(an assimilation that implies, at best, a complete subordination of the philosophical to the mathematical).

Cavaillès' critique in the passage from *Sur la logique et la théorie de la science* quoted above is particularly allusive and synthetic. However, it is possible to distinguish two steps: a warning against the "tentacular" dimension conferred on mathematics by the Brouwerian definition, and the fact that this definition is, so to speak, counter-productive, because by instrumentalizing mathematics too widely ("putting the world in order"), it establishes dependencies within it. Cavaillès first points out that the Brouwerian approach makes mathematics difficult to distinguish from other spheres of knowledge. In particular, the specificities of physics are "drowned out" by this insufficiently critical characterization. But it doesn't stop there. By making mathematics an instrument for ordering the world, not only does it encroach on other sciences (forcing it to take on extrinsic elements and processes), but it also subordinates it to a function and to what it is the instrument for: it then becomes dependent on, and must be able to account for, the relationship to the "world" (which it organizes).⁽²¹⁾ To put it briefly: seeing mathematics as "the ordering of the world" presupposes a world to be organized — and thereby establishes a relationship of dependence (on the world, in this case) that the intuitionist doctrine aimed to banish. Indeed, it was a form of creative independence that the proponents of the Brouwerian perspective wanted to establish. Cavaillès of course fully endorses such a desire, but he never ceased to underline the relationships of dependence established by the intuitionist claim to independence. Already in *Méthode axiomatique et formalisme*, he argued that this independence was ultimately based on dependence on an extrinsic intuition (which he identified as "a remnant of attachment to the logical *a priori*").⁽²²⁾

But is Cavaillès' own position exempt from such criticism? How can this "independence" be conciliated with his more general project of understanding thought? With this question, we return to the problem posed by the notion of "thought". Although present throughout the work, it is never tackled head-on, and the same is true of its answer: always underlying, it develops, so to speak, in parallel with the work of elucidating the mathematical. For the

⁽²¹⁾In Wittgenstein's aforementioned terms, we could say that Brouwer makes "the mistake of explaining royal dignity from the king's utility".

⁽²²⁾*Méthode axiomatique et formalisme*, p. 189.

moment, it's clear that it requires a deeper understanding of the notion of "independence".

The autonomy of thought. Clearly, the term "independence" refers to the autonomy-interiority binomial already mentioned several times. Once the link has been made with the project to investigate the nature of thought, we arrive at a characterization of mathematics as marking the autonomy of the "attribute" of thought: mathematics as thought in its purest expression, subject to objects and gestures of its own making. Independence means freedom from all external tutelage, from any "order" other than that of thought. Mathematics is thus driven solely by the structuring force of the intelligible, and not by a relationship to the "real world". However, this does not mean that mathematics is outside the "world", nor that it cannot "organize" the sensible; rather, it means that mathematics should not be seen simply as an intermediary between the mind and "reality". If there is such a thing as "mathematical experience", it is not in the sense of a dualistic confrontation between subject and object, but of a claim to autonomy for the mathematical process.

"What do you call the real world? I'm not an idealist, I believe in what is lived. To think a plan, do you live it? What do I think when I say I think this room? Either I'll speak of lived impressions, rigorously untranslatable, rigorously unusable by means of a rule, or I'll make the geometry of this room and I'll do mathematics," asserts Cavallès (in response to Maurice Fréchet) in his 1939 lecture at the Société française de philosophie.⁽²³⁾ His thinking may well have evolved on this point (Cavallès still subscribed at this time to the idea of a solidarity of successive mathematical "gestures" with the "primitive sensible"),⁽²⁴⁾ but it is nonetheless notable that the alternative evoked combines criticism of a directly subjectivist perspective with rejection of a dichotomy that frontally opposes, and subordinates, mathematics to the sensible. Such a dichotomy tends to give mathematics the role of *medium*, and thereby contravenes the autonomy of its becoming. It is this latter perspective that will be pursued and extended in the posthumous work — as in the warning addressed to Brouwerian intuitionism.

⁽²³⁾In *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 625.

⁽²⁴⁾Cf. *Méthode axiomatique et formalisme* in *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, pp. 186 and 187.

The mathematical expression of a thing is of an order distinct from that of that thing. But this does not mean that it is constructed in contradiction to it: it is simply governed by rules of its own, and therefore does not strictly speaking depend on the “real” (even if it expresses it). As Wittgenstein points out, also refusing in his own way to set up a heteronomous framework to account for the mathematical process: “Even if the demonstrated mathematical proposition seems to point to an external reality, it is only the recognition of a new measure (of reality)”.⁽²⁵⁾

In this way, Cavallès avoids the problem of the “emptiness” of mathematical propositions, which the idea of autonomy could give rise to: for they are not like “boxes” into which the objects of the world must fit, but of another order than these. The fact that mathematical experience is of a different order from that of physics does not mean that it is cut off from the world. There is indeed the idea of a possible convergence, or even of a common denominator between the two orders, but this does not call into question mathematical autonomy: there can be no validation of one order by the other. Thus, the autonomy we are discussing clearly expresses the rejection of a mathematical thought whose *legitimizing function* would be to represent the real world (i.e., to enter into a point-by-point correspondence with a “reality” to which it would be frontally opposed), while not excluding the idea of coincidence.

This search for a path which, without denying the sensible inscription of the mathematical process, aims to respect its autonomy, could be seen as part of the Spinozist approach shared by Cavallès and Wittgenstein. But it is in the foundation of this perspective that the divergences between these two authors are revealed, or more precisely in the fact that for one, mathematical autonomy refers to an underlying intelligible structure, the modalities of which are identifiable, while for the other, there is nothing below this autonomy — which must not be hindered (by burdening it with inadequate and extrinsic representations). In both cases, there is a detachment from dualist and idealist positions; but in Cavallès’ case, this is part of a more pronounced progression towards Spinoza. Indeed, while mathematicians do not “discover”, they do realize virtualities necessarily inscribed in conceptual development, thus revealing the “internal links of ideas”. These “essential sequences” coincide with mathematical

⁽²⁵⁾ *Remarques sur les fondements des mathématiques*; Part 3, § 27.

experience as a regulated system of gestures and acts.⁽²⁶⁾ Cavallès is therefore close to Spinoza when the latter states that "it matters little when it comes to figures and other beings of reason" that the sequence of ideas reproduces in the understanding the sequence of nature : what matters is precisely this privilege accorded to the order and sequence of ideas, characteristic of all thought, and affirming the active power of the understanding:⁽²⁷⁾ what is important here is precisely this privilege accorded to the order and sequence of ideas, characteristic of all thought, and affirming the active power of the understanding. In this sense, in Spinoza, as P.-F. Moreau puts it, "(...) experimentation [in its "physical" sense] serves to decide not between true and false laws, but between true laws that are effective in a given case and those that are not".⁽²⁸⁾ Of course, the Cavallèsian position, informed by the historical evolution of the two sciences — physics and mathematics — and by the development of formalizations, would be different, more concerned with the specificity of physical science and the new issues associated with technique and the experimental process. However, to underline this connection with Spinoza, let's recall what Cavallès said in 1936 in a letter to Lautman: "Basically, I think we agree more and more — nothing distinguishes a mathematical or physical axiomatized system outwardly. Remains the function with reality: in mathematics, experience is exhaustively described by axioms; thought is distinct from reality only by the arbitrariness it recognizes in the combination of axioms. In physics, we return to the classical meaning of experience — approximation — this is still not very satisfactory." In this respect, the most complete approach to the problem of "the intersection of these two processes" — physical experience and mathematical experience — of "essences"⁽²⁹⁾ different, is found in the posthumous work at the very end of the second part: "The true experimental process [in the physical sense] is elsewhere, in the aims, uses, and actual constructions of instruments, the whole cosmic-technical system where its meaning is revealed and whose unity as well as its relation to the autonomous

⁽²⁶⁾ Even if the Cavallèsian notions of "gesture" and "act" do indeed reflect a concern for the sensitive part of the mathematical process, they are to be grasped on the side of science itself, of its autonomous dynamism.

⁽²⁷⁾ *Traité de la réforme de l'entendement* ; § 95.

⁽²⁸⁾ Cited by F. Audié in *Spinoza et les mathématiques* (Presses de l'Université Paris-Sorbonne, Paris, 2005), p. 93.

⁽²⁹⁾ *On Logic and the Theory of Science*, p. 54.

mathematical unfolding pose the fundamental problem of physical epistemology.”⁽³⁰⁾ And a little further on (in the section developing the critique of logicism): “Now, the descriptive intention of physics transforms everything: not only [because] the object becomes determinant, the starting point for theories, the reference point for their results, but also by virtue of the subordination of mathematics as a guiding instrument to the formal sequences themselves. Undoubtedly, the internal necessity of their development remains, and the possibility of a choice between them gives them the same independence from what exists as the understanding had from the will in the Leibnizian divinity. But it’s still a question of virtual existence, and therefore of a necessary affinity between these sequences and the characteristics of the existing object.”⁽³¹⁾ With this idea of a “necessary affinity” between mathematical virtualities (the “beings of reason”) and the physical existing object (the “physical and real beings”), we find a type of Spinozist connection that preserves the autonomy of the order of thought, while allowing for the possibility of its correlation in the order of nature.⁽³²⁾ In any case, and whatever Cavallès’ “definitive” position on physical science (a question which it was not his intention to tackle head-on, and whose answer can therefore only be guessed at from the analysis of mathematical thought), it appears that Cavallès is in line with Spinoza when the latter retains from mathematics the necessary and intrinsic causality of a sequence of ideas. In this respect, we can refer to the study by F. Audié, in which, to characterize the “asceticism” common to Cavallès and Spinoza, he quotes J.-T. Desanti (speaking of Cavallès) and brings together various formulas all having to do with this self-engendering of concepts and structures:

“This entailed a whole asceticism: he had to learn to discipline himself to speak only from within.” J.-T. Desanti’s use of the term “asceticism” is therefore mainly linked to his reading of *Méthode axiomatique et formalisme*: “it’s this same Cavallès that I find again. Entirely caught up in the exigency of his object; entirely faithful to its necessity; *noësis noësiôs* in a certain sense: thought of what mathematics produces explicitly according to its sequences”, to

⁽³⁰⁾ *Ibid.*, p. 55.

⁽³¹⁾ *Ibid.*, p. 56.

⁽³²⁾ Cf. the famous proposition 7 of the second book of the *Ethics*, founding the Spinozist “parallelism” (to use Leibniz’s expression): “The order and connection of ideas are the same as the order and connection of things.”

Cavaillès' words reported by G. Canguilhem: "Whatever the importance of the suggestions of physics for the position of new mathematical problems and the construction of new theories, the authentic development of mathematics under the accidents of history is oriented by an internal dialectic of notions", and to H. Sinaceur's comment: "It is of course to Spinoza that we must relate Cavaillès' conceptual automatism (...). For Spinoza, the idea refers neither to a subject nor to an object, but to another true idea."⁽³³⁾

We also find in Wittgenstein this idea of an "asceticism" that would be an endeavour to speak only "from within". But for him, it would also mean an "asceticism" of the philosophical itself, i.e., a resistance against philosophical tendencies. And while Cavaillès does indeed offer this dimension of resistance (criticism of dichotomy, of *a priori* dogmatism, of the philosophies of constituent consciousness, etc.), it is linked to an immediate rehabilitation of philosophical discourse via the guiding project of investigating the nature of thought. In this respect, the interpretation of the idea, common to both authors, of the complete independence of science is particularly revealing. While Wittgenstein's autonomy of mathematics is associated with a fairly radical critique of the philosophical, Cavaillès, on the contrary, finds in this same autonomy (which presupposes, indeed, a critical passage through a denunciation of the interference of the philosophical in the mathematical) a richness for philosophical discourse: mathematics as a source of lessons on thought.

In the same way, and to return to the Brouwerian idea of "free creation", it is possible to consider that this idea could lead (independently of Brouwer's own subjectivist perspective) to a critique of philosophy's claims to theorize the essence of a process. What we mean by this is that it seems difficult, outside a "solipsistic" framework, to maintain simultaneously the idea of a mathematical "free creation" and the general theorization of the latter: and yet this is Cavaillès' challenge. However, Cavaillès remains constantly cautious, and even rather evasive, both on this particular point and, more generally, in his analysis of "epistemological philosophies of immanence" (by which he means Brunschvicgian and Brouwerian

⁽³³⁾F. Audié, *Spinoza et les mathématiques*, Presses de l'Université Paris-Sorbonne, Paris, 2005, p. 113.

thought). Indeed, he opens his remarks by stressing that "it is perhaps preferable to adjourn consideration of these philosophies until the systematic development of scientific epistemology. The notions they invoke are too closely linked to the development of science to be clarified in the course of a simple point-of-view survey."⁽³⁴⁾ We can now regret this "adjournment", which has deprived us of a detailed analysis of a thought that ultimately presents important similarities with his own, as well as, correlatively, of his precise position regarding the problem he raises, in passing, in Brouwer. Incidentally, the allusive nature of Cavaillès's remarks on this point may partly explain why he has sometimes been associated with the Bourbakist group, which, while not a continuator of Brouwerian intuitionism, nevertheless had an idea of philosophy as subject to mathematical exactitude (this group was formed at the same time as Cavaillès was at the Ecole Normale Supérieure, and several of its members — some of whose work appears in our author's two dissertations — were friends of his). To express our disagreement with this rapprochement, we can refer to Hourya Sinaceur's formula in her introduction to *Lettres inédites de Jean Cavaillès et Albert Lautman*:⁽³⁵⁾ "His friendship with the Bourbakists did not prevent him from refusing to subordinate philosophy to mathematics. He saw mathematics rather as an instrument of rigorous thought, not thought itself ; an 'experience' that encourages reflection, but does not dispense with it."

§ 2. — The instrumentalization of mathematical thought (second danger).

But this brings us closer to the second danger mentioned in the introduction. Indeed, might we not consider that to make mathematics an instrument of knowledge of thought — a thought with which it would not be confused — is to assign to it a particular function that is not directly related to its autonomy, contravening its "independence"?

Here again, the answer is obviously negative, but here too, the answer is to be found in the implicit, or more precisely, in what is revealed by the immanent workings of Cavaillès' thought. The

⁽³⁴⁾ *Sur la logique et la théorie de la science*; p. 30.

⁽³⁵⁾ *Revue d'histoire des sciences*, 40-1, 1987, pp. 117-128. This publication is part of a thematic issue: *Mathématiques et Philosophie: Jean Cavaillès, Albert Lautman*.

difficulty lies in the fact that this very workings prohibits any form of non-mathematical definition of mathematics, any “way out” of mathematical autonomy.⁽³⁶⁾ It is therefore via mathematical immanence, in a “self-illumination” of the latter, that the immanence of thought appears in negative. In other words, bringing the concept of “thought” into play is not incompatible with a refusal of any definition or interpretation outside mathematics: it simply shows what is at work *in* mathematics, without prejudging its own development.

This latter problem is not to be found in Wittgenstein, whose rejection of any *a priori* and extrinsic interpretation, linked to a virulent critique of “classical” philosophical discourse, is manifested in his exclusive study of the “grammar” of the word “thought”, associated with the logico-mathematical order alone (and not with philosophy). In a way, Wittgenstein chooses to stop the investigation at the logico-mathematical level, without seeing in it the distinct expression of the immanence of thought: as we indicated above, there can be no “theorization” of mathematics for him, which would make it possible to establish the general “structure” of science. In this sense, his approach is more descriptive, deliberately focusing solely on the pure effectivity of a fact of language (in this case, the mathematical “game(s)”). Does this mean that Cavallès’ approach requires a greater philosophical “investment”? Yes, but only insofar as his position aims to reconcile this rejection of any *a priori* dogmatism with that of a total identification of mathematics with thought. This is, in fact, the minimal assumption for anyone wishing to keep philosophy — despite the criticism to which it is subjected — within the realm of thought (which is not the case with Wittgenstein). We agree that such an assumption, which takes a Spinozist form in Cavallès’ work, is a rather limited philosophical requirement — but a decisive one nonetheless.

Both Spinoza and Cavallès think “from within”, which for them is the mark of the same subjection to the order of thought, to the necessity of its intelligible sequences: the unfolding of mathematics is immanent because the unfolding of thought is immanent. And this is why, even before he has begun to clarify the nature of mathematical thought, Cavallès’ method is already to think from within (the introduction to the secondary dissertation is explicit in this

⁽³⁶⁾As evident in his critique of intuitionism, Cavallès is wary of the instrumentalization of mathematical thought, which endangers the independence of an immanentist theory of science.

respect): in this, he is already a Spinozist. A kind of circle? Yes, but this is not to be confused with a *a priori* interpretation. Rather, it is a "dialectical" demonstration (the term being taken in the same sense as when it designates mathematical development) of the superfluous nature of speculations on the subject, and the subject/object relation, for the elaboration of a theory of science. In this respect, the presence in the conclusion of *Axiomatic Method and Formalism* of Husserlian themes still playing a decisive positive role is flagrant proof that not everything is settled from the outset in Cavailles, and that his thought proceeds dialectically, by successive evaluation and overcoming of guiding theoretical hypotheses.⁽³⁷⁾ In a way, we should not regard as an *a priori* the fact of refusing all *a priori* — and thus sticking to what mathematical becoming expresses by itself. As for the fact that this becoming is not exclusively confused with thought (even if it is a privileged expression of it), this is a hypothesis that in no way detracts from mathematical immanence, but is a matter for philosophy alone — and, more precisely, for the possibility of safeguarding "classical" philosophical discourse, i.e., that which is conceived as the relative of mathematical discourse by thought. Philosophy and mathematics do not fully coincide in thought, but they are nonetheless two thought experiences, guided by the same intelligible necessity. It is still a matter of following, and even embracing, the mathematical movement, so as to grasp it reflexively from within. And philosophy lies precisely in this reflexive process that immerses it in the mathematical movement, and at the same time distinguishes it from it.

In each case, a constitutive property of *the essence of thought* — or of intelligible sequences — is manifested: *the paradigm and the thematic*.⁽³⁸⁾

Here, Cavailles clearly isolates, through mathematics, two major characteristics of the general functioning of thought: a good

⁽³⁷⁾The conclusion of the main dissertation reveals a hesitation, and an attempt at conciliation, between a Spinozist influence "patronizing" (to use Cavailles's own expression in the letter to his father in which he reports on his defense) the autonomy of mathematical experience, and an analysis in very Husserlian terms of "the enlargement of consciousness". The posthumous text, on the other hand, which is probably the outline of a larger work that Cavailles sometimes refers to in his correspondence as "*L'expérience mathématique*", closes significantly with a critique of the Husserlian perspective.

⁽³⁸⁾*Sur la logique et la théorie de la science*, p. 41. Cavailles italics only "paradigm" and "thematic".

example of the philosophical utility of mathematics. The formulation itself is interesting, as it underlines that criticism of foundational undertakings and *a priori* definitions is not incompatible with *a quest for essence*. This is the precise point of divergence with Wittgenstein, who accompanied Cavaillès throughout his critical approach to the characterization of science: for the author of *Philosophical Investigations*, the desire for foundation, like that for exhaustive definition, is confused with the pathological — and typically philosophical — search for essence. There is therefore no reason for the latter to uncover a theoretical framework in the scientific movement that would merge with the very progress of thought, even if this framework were obtained by “self-illumination”. Mathematical language “is in order, as it is”,⁽³⁹⁾ which means that its movement does not reveal — nor even “self-reveal” — anything other than itself; it cannot therefore be authentically captured in a more global or structuring perspective, and while it is indeed “necessary”,⁽⁴⁰⁾ it is not subject to a pre-existing necessity whose “principle” could be isolate. Thus, in contrast to their similar characterizations of science,⁽⁴¹⁾ Cavaillès’ desire to “dig deeper” into mathematical immanence itself, which he sees as the stage for thought’s own necessity, is clearly evident. In this respect, the last formula quoted sheds retrospective light on what Cavaillès meant, on page 39 of the same work (where he poses the problem of the status of the theoretical discipline), by the “permanent emergence” of the statements of a doctrine of science. He was referring here to the dual process of “paradigm” and “thematization”.⁽⁴²⁾ One distinguishing the demonstrative forms by which a field of objects is extended (we find the “idealization” or “addition of ideal elements” of the main dissertation), the other the systems of links or relations by which “operations” become objects of a new field (“...thematization proper : transformation of an operation into

⁽³⁹⁾ *Investigations philosophiques*, Gallimard, Paris, 1990, §98.

⁽⁴⁰⁾ A characterization that relies on an analysis of the status of the mathematical “rule” (analysis which it is not the place here to detail).

⁽⁴¹⁾ Autonomy, unpredictability and necessity could be seen as the three *leitmotifs* of their analyses.

⁽⁴²⁾ For an in-depth analysis of this dual process (and a fruitful perspective on category theory): see the book *Pour Cavaillès* (Pont 9, 2021), by C. Houzel, D. Nordon, X.-F. Renou, H. Roudier and J.-J. Szczeciniarz (in particular chapter 3 “Cavaillès, Spinoza”, and more particularly pages 69 to 81).

an element of a higher operative field").⁽⁴³⁾ "The process of separation is twofold: longitudinal, or coextensive with the demonstrative sequence, and vertical, or establishing a new linking system that uses the old one as a starting point (...)."⁽⁴⁴⁾ This clearly reveals the essential unity of thought "in act", i.e. equally (and without any contradiction between the terms) progressive and indefinite: "the idea of the idea manifests its generative power on the plane it defines without prejudice to unlimited superposition."⁽⁴⁵⁾

§ — Conclusion.

Our study thus leads to the formulation of an alternative for these immanentist investigations concerned not to interfere with mathematical autonomy: either we follow the Wittgensteinian path, or the more directly Spinozist one of Cavailles. In other words:

1. To accept the hypothesis of a self-revelation of the structure of science; in which case, we can "save" the possibility of philosophical discourse — understood "classically" as an activity of thought in the same way as mathematics.
2. Refusal of this hypothesis, and therefore condemnation of such a conception of the philosophical.

A word of clarification is required regarding our characterization of the Cavaillesian path. It might seem surprising to refer to Spinoza in order to contest the complete identification of thought with mathematics, and thus to maintain, within thought, a certain distinction between philosophy and mathematics. These disciplines were not as distinct in Spinoza's time as they are today; and, in this respect, it would certainly be possible to argue that, according to an expanded conception of "mathematical", Spinoza did indeed consider himself to be a mathematician. Ethics itself could be seen, from this less clear-cut perspective (where the boundary between mathematics, philosophy and "science of nature" is particularly porous), as the *mathesis* of a part of nature. Yet, even if he is

⁽⁴³⁾ *Méthode axiomatique et formalisme in Oeuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 177.

⁽⁴⁴⁾ *Sur la logique et la théorie de la science*, p. 41.

⁽⁴⁵⁾ *Ibid.*, p. 46.

often reluctant to establish delimitations, Cavailles does make these disciplinary distinctions; but he sets out to understand and characterize the *mathesis* — or the “intelligible sequences” as he puts it⁽⁴⁶⁾ — through its privileged expression (mathematics).

In fact, a “philosophy of mathematics” or even a problematization of the insertion of the philosophical in mathematics would have made little sense to Spinoza. But mathematics, understood as a model of demonstrative method, nonetheless plays a primordial role in his system. It’s not demonstrative science as such that Spinoza is concerned with, but rather the way in which its order is inscribed in knowledge of the world (and, in the case of ethics, in human action) — and he uses the mathematical example for this. Recourse to the mathematical example puts into perspective the value for knowledge of an immanentist and necessary approach to reality.⁽⁴⁷⁾ Whether in the *Treatise on the Reform of the Understanding* (§22 to 25) or the *Ethics* (second scolie of proposition 40 in Book II), it’s always the mathematical model that provides Spinoza with an example of the last — and highest — genre of knowledge. This means that mathematics is a privileged path (if not the privileged path) to an adequate idea of what true knowledge is. Does this mean that, rather than an example, mathematics is the paradigm of all thought? Not quite: mathematics is not a model for thought itself, but rather for us who want to understand thought. Thought manifests itself in each of its “expressions”, and if mathematics is favoured by us, it’s perhaps because it’s purer, clearer (not having, as Spinoza says, to concern itself with “real, physical beings”), but not because it alone belongs to the true movement of knowledge. Thought does not function differently according to the domain in which it is exercised; simply, the immanence and necessity of this functioning is likely to appear to our eyes with greater or lesser clarity. Hence Spinoza’s recourse to mathematics, which makes it easier to grasp “what it means to know, to think”.

The “asceticism” of which Desanti spoke to characterize Cavailles’ method is thus revealed in a new light. For it is no longer simply a question of the effort — typical of his original

⁽⁴⁶⁾This formula, which Cavailles affixes to “thought” and which thus makes explicit the meaning he gives to it (cf. *Sur la logique et la théorie de la science*, p. 41), is reminiscent of the Spinozist “*concatenatio*” of ideas.

⁽⁴⁷⁾“If men knew clearly the whole order of nature, they would find all things as necessary as all those dealt with in Mathematics” (*Pensées métaphysiques*, Part 2, chap. IX).

method — to remain within mathematical immanence, but of going beyond it to reach, through it, the immanence of thought in general. Interestingly, Cavailles himself uses the term “asceticism” in his discussion of “epistemological philosophies of immanence”: “(...) the terms spirituality, immanence, presuppose the possibility of an asceticism or a deepening of consciousness other than scientific understanding alone.”⁽⁴⁸⁾ “Asceticism” is therefore not just an image transposing the irreducible elements of the Cavaillesian approach — interiority and necessity — into the ethical field. Nor is it to be understood solely in the attenuated sense of a particularly strict mental rule. It is properly self-absorption, submission to a creative necessity — Cavailles thus joins Spinoza in what he expresses *ad usum vitae*: “to expect and endure with an equal soul the one and the other face of fortune: because everything follows from the eternal decree of God with the same necessity as, from the essence of the triangle, it follows that its three angles are equal to two rights”.⁽⁴⁹⁾ Immanentism in Cavailles — as in Spinoza — thus goes beyond the sole development of mathematics, and reveals the latter’s connection to a necessity that, in a way, pre-exists it: that of the order of thought.

From this point on, we can resolve the apparent riddle that philosophy can “profit” from mathematics, provided it respects its perfect autonomy (and therefore does not interfere in its own development). This may seem paradoxical, but it’s actually quite convincing, since it’s only by refusing to dictate its own laws to mathematical development that philosophy can understand the true workings of thought, and find its own place within them. An observation that is not, after all, an end in itself, but rather a promise of future fruitfulness. Didn’t Cavailles say, at the end of his two dissertations: “Now I’ll be able to work”?⁽⁵⁰⁾

⁽⁴⁸⁾ *Sur la logique et la théorie de la science*, p. 34.

⁽⁴⁹⁾ *Ethics*; second part, proposition XLIX, scolie (end).

⁽⁵⁰⁾ In Gaston Bachelard’s afterword to Gabrielle Ferrières’ book: *Jean Cavailles – un philosophe dans la guerre*; éd. du Félin, Paris, 2003.

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