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Gilles-Gaston Granger's style and duality

Éric Audureau[(1)]

Abstract. After a brief presentation of the original features of Granger's philosophical system, we propose to show that the notion of mathematical style should not be seen as a substitute for the traditional philosophy of mathematics. To this end, we first show how style depends on the principle of duality. We then try to show, by examining it through the sieve of progress in mathematical logic, that the principle of duality does not keep its promises.

Keywords. Contemporary French epistemology; logic and sequent calculus; philosophy of mathematics; principle of duality; mathematical style; philosophical systems.

(1) The material for this article comes from two lectures: "M. Granger et le calcul des propositions", given in December 1990 at the Séminaire d'Épistémologie Comparative de l'Université de Provence, and "Dualité et contenu formel", presented at the *Séminaire Granger*, organised by Gabriella Crocco and Philippe Abgrall, on 22 November 2022.

To present in a few pages the philosophy of Gilles-Gaston Granger (1920-2017), Comparative Epistemology, and to describe what is specific about it, in particular in his epistemology of mathematics, is a challenge that it is better to admit that we are incapable of meeting, if only because we are far from being one of its exegetes. We have therefore chosen a stratagem that will allow us to sketch the main aspects of this philosophy in a somewhat disjointed way, starting with a fact that is present on the current philosophical market.

With the notion of style, Granger's work seems to be enjoying a revival of interest today, particularly, but not exclusively,(2) due to its role in the philosophy of mathematics. This attraction of contemporary thought toward what, until recently, proponents of analytic philosophy disdainfully called "French-style epistemology" is undoubtedly comforting. However, this new craze, which, as far as we can tell, is in line with current trends in the philosophy of mathematics that favour the study of the "work of mathematicians" to the detriment of research into the foundations, should not be based on equivocation.(3) However, we would be lending Granger too little if we thought that the notion of style was autonomous, detachable from his thought, and ready for use like a self-service tool, like a formula in a repertoire of algebraic expressions.

According to Granger, style cannot be conceived independently of a *primitive category of thought*, which he calls the *principle of duality of operation and object*, and which we will try to show is the keystone of his philosophical system. Now, once this link has been made explicit, Granger's epistemology finds itself on much more familiar ground, since it has dominated the philosophy of knowledge of the twentieth century: that of the controversies over the nature and origin of mathematical knowledge and its relationship with logic.(4) We would be lending the style too much credit if we were to see it as a substitute for the classical philosophy of mathematics as it emerged at the beginning of the twentieth century, when *in fine* it is merely a constituent of a new philosophical system that is being proposed to the debates on the foundations of mathematics.

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(2) For example [Costa 2013].

(3) In this respect, the pages of the *Stanford Encyclopedia of Philosophy* which set out to show that Granger's epistemological role of style is compatible with a realism of mathematical entities or structures are so ominous that one wonders whether it was really the *Essai d'une philosophie du style* that their writer had read.

(4) These controversies died out with Quine's death. Following this, avatars of Quinean naturalism, such as Nancy Cartwright's, flourished to invade the field of the philosophy of knowledge.

Granger's position, because of its indisputable originality, inevitably gives rise to a diagnosis. We have therefore taken the risk of formulating one in the terms chosen by the author. It would appear that the keystone of the edifice is quite fragile, compromising, among other things, the solidity of one of its pillars: style.

§ 1. — **Overview of the Granger system.**

The major works that brought him fame(5) could hardly reveal Granger's real ambition to his readers. These works, principally *Pensée formelle et sciences de l'homme* (1960) and *Essai d'une philosophie du style* (1968), deal only with certain elements of his system, without setting it out, let alone professing it. The profound originality of the subject matter of these essays was enough in itself to attract attention from both the scientific and philosophical communities. Moreover, it seems that Granger never set out his philosophy systematically, although the main themes are to be found more or less explicitly in each of his books. No attempt will be made to fill this gap, if indeed this is one. However, it seems useful to give, as a preliminary, a rough description of the most general aspects of this philosophy, in particular those which will not be discussed in these pages, so that the reader unfamiliar with Granger's thought could form a provisional image of it.

We shall omit from this description the influences acknowledged by the philosopher. Let us mention them nonetheless, following the author in what is undoubtedly, along with his *Leçon inaugurale au Collège de France*.(6), the most accessible and concise account of his work(7) He recognises four main influences. The "sometimes opposing" influences of G. Bachelard and J. Cavaillès; the influence of "Martial Gueroult's exegesis of philosophical systems"; and, finally, the influence of the "great Viennese of the 1930s" with whom, and against whom, he tried to set out the problems of science.

(5) However, this notoriety never crossed the boundaries of academic research to reach the public arena, as demonstrated by the fact that the essayist Élisabeth Badinter and the politician Robert Badinter published a work on Condorcet while being unaware of the existence of *La mathématique sociale du marquis de Condorcet*, which Granger published in 1955.

(6) [Granger 1987].

(7) [Granger 1985].

A complete philosophical system. Firstly, "any philosophy is [...] a system, in an apparent or hidden way, whether it is openly exposed or claimed as such, or whether, on the contrary, the very idea of a system is explicitly rejected".(8) Consequently, for example, a philosophy of mathematics that was only a philosophy of mathematics would not be philosophy in the strict sense of the term, as Granger understands it. Among other things, it would evade the problem of the unity of science. A problem that must be "maintained [because] there is indeed a unity of science, perfectly compatible with the diversity of its manifestations and methods".(9) So we can see the difference with the neo-positivist conception, which tried to reduce all scientific knowledge to a single, general method. Moreover, the systematic form of any philosophy worthy of the name seems to link it to scientific knowledge. Nevertheless, against Kant, who "oriented modern epistemology, if not in its content, at least in its form, by questioning the possibility of science",(10) Granger rejects out of hand the idea that philosophy can be a science, although it is a question of real knowledge proceeding by concepts, but knowledge without objects.(11)

Secondly, this philosophy is complete in the sense that Granger has spoken out on all the important questions dealt with by philosophy from its origins to the present day. This statement may come as a surprise, given that his work, which consists either of works on epistemology or monographs on leading authors such as Aristotle and Wittgenstein, seems to be silent on ethics and moral, even though they are part of the programme of all the great authors of the past. Granger has explained this silence. For him, ethics and morality inevitably generate affects. It is true that philosophy can "*enlighten the choice of a guide for action*. In this sense, every Moral has a purely philosophical part and a post-philosophical part, but it is only the latter that formulates precepts for action".(12) Philosophy only penetrates politics or morality through ideologies, i.e. by transforming concepts into images and substituting spontaneous affective movements for deliberate of thought, because that

(8) [Granger 1993, 390]. See also [Granger 2003, 13].

(9) [Granger 1988, 123].

(10) [Granger 1960, 8].

(11) The hurried or lazy reader who wishes to avoid reading [Granger 1988] and learn more will find a concise description of the differences between science, aesthetics, and philosophy in the concluding article [Granger 1994], "Formes, operations, objets dans les sciences et en philosophie".

(12) [Granger 2003, 17]. See also [Granger 1985, 198].

is the price of action. This position fits in very well with the anecdotal fact pointed out by Wikipedia: Granger defined himself as a civil servant of reason. The philosopher is an employee of the State who has duties towards the Nation. As a citizen like any other, his opinion on the laws governing the City counts for no more than that of any other citizen.

The role of language. "We do not notice enough how the consideration of linguistic formulation has been silenced in Kantianism".(13) Here Granger adopts a largely consensual position, relentlessly cultivated by analytic philosophy, but his conceptions are of an entirely different tenor. A few examples of the roles he lends to language, or rather to different types of language, will show this.

1.0.1. The languages of science. A science cannot be constituted without the creation of its own language. The example of the transition from alchemy to chemistry, dealt with in five admirable pages in *Formal Thought and the Human Sciences*, shows this clearly.(14) What's more, the symbolism *of the written* languages of science makes it possible to break free from the one-dimensionality of ordinary language. When the idea of valency replaced that of electro-positivity, another aspect of chemistry's symbolic language emerged — the spatial arrangement of nomenclatural symbols — alongside the nomenclature developed by Guyton de Morveau and Lavoisier. This symbolic property is highlighted by the notion of isomerism "which allows distinct chemical and physical properties to be represented by the different arrangement of signs in a two- or three-dimensional space".(15) The mathematician reader will readily agree with this, recalling the many services rendered to algebra by two-dimensional symbolism.

We can see that, with the creation of the specialised languages of science, the problem of the unity of science is proving to be more complex than Carnap had envisaged, and that it needs to be posed afresh.

(13) [Granger 1960, 12]. The question was taken up again and explored in greater depth in [Granger 1979, 24-35].
(14) [Granger 1960, 45-50].
(15) [Granger 1960, 50].

1.0.2. The merits and limits of mathematical linguistics. Granger paid particular attention to the progress, and possible missteps, of linguistics, which he regards as the most advanced of the human sciences.

One example of progress is the importance attributed to the structural method in linguistics, advocated by Baudoin de Courtenay and Saussure, culminating in Troubetzkoi's phonology. This structuralism, clearly distinct from Bourbaki's, nevertheless illustrates the fruitful nature of the approach, common to linguistics and mathematics, which amounts to considering that "the object is graspable in its depth not as the bearer of internal (...) properties but as a system of relations between otherwise unmarked elements, whose only envisaged properties derive from these very relations".(16).

As far as misunderstandings are concerned, we should mention the precise and rigorous elaboration of the distinction between a *symbolic system* and a *formal system*, designed to dispel the abusive identification of natural languages with formal systems such as those of logic and mathematics, either by treating a natural language as a formal system or, conversely, by viewing a formal system as equivalent to a natural language.(17)

1.0.3. The search for universals.(18) The foregoing clarification should not lead us to believe that for Granger logic and linguistics need not be in any way related. As we shall see below, the principle of duality is seen as the common origin of logic and language.

If linguistics limited itself to a description or explanation of the facts of given languages, it would "only half deserve the name of science". In order to achieve the status of a science, so that linguistic facts become objects of knowledge, linguistics cannot dispense with the search for the universals of language. This is the price to be paid if the regularities observed in the description of particular languages are to escape *ad hoc* explanations that have no general scope. After pointing out the illusory nature of the search for a mother language that would be the repository of universal categories and the failure of the Leibnizian project, acknowledged by Leibniz himself, to isolate a logical core common to all languages, Granger examines in more detail various attempts at an 'empiricist census' of universals: those

(16) [Granger 1960, 2].
(17) The article "Langage et système formel", which appeared in 1971, is reprinted in modified form as the fifth chapter of [Granger 1979]. It was reprinted in its original form in [Granger 2003].
(18) [Granger 1979, ch. IX].

of Charles Hockett, Joseph Greenberg and, with particular attention, René Thom,[19] and rejects them. Thinking that he had shown that any attempt at an inductive census of universals was futile, he then formulated his own proposals, defining along the way what a linguistic theory should be. Thisis worth noting as something of an anomaly, since in principle, comparative epistemology is expected not to encroach on the domain of science.

We need hardly add to these remarks that the second part of *Essai d'une philosophie du style* is devoted to *Style and the structures of language* in order to understand that Granger's remedy for Kant's silence on language can in no way be equated with what analytic philosophy calls the "linguistic turn". Indeed, Granger's approach to language constitutes an explicit departure from analytic philosophy.

Philosophy for the Age of Science: Comparative Epistemology versus Comparative Metaphysics. To conclude this summary description of Granger's philosophy, it is appropriate to say a few words about the project of a Philosophy for the Age of Science. This enables us to highlight three important points: first, the influence that Granger may have had on subsequent developments in French philosophical research; second, some of the distinctive features of this research; third, what unites and opposes Granger to his companion in thought, Jules Vuillemin.

1.0.4. In 1968, the year of questioning, Granger and Vuillemin created a collection of books entitled *Philosophie pour l'Âge de la science* (*Philosophy for the Age of Science*) and the eponymous journal, *L'âge de la science* (*The Age of Science*). The scientific committee included N. Chomsky, G. Kreisel, W. Quine and R. Thom. This all indicates that this project was primarily concerned with the origin, nature and function of mathematical knowledge. More generally, its main aim was to bring French philosophy out of a form of provincialism that had kept it away from the currents that, between the two world wars, had inspired the Germanic and Anglo-Saxon worlds. Before these initiatives, before the work of Granger and Vuillemin, the study of the works of Carnap, Frege, Goodman, Quine, Russell and Wittgenstein was absent from the French philosophical landscape. However, this movement towards the 'internationalisation' of French philosophy did not abolish the specificity of the epistemology promoted by Granger and Vuillemin.

(19) Granger dwelt on Thom's linguistic conceptions on several occasions.

1.0.5. Three main features characterise this project. The first, inherited from the work of Martial Gueroult, who introduced the stuctural method into the history of philosophy, consists in severing the links that indecisively unite this history with that of ideas.(20) The philosophical systems of the past must, to use Saussure's expression, be studied in themselves and for themselves. This amounts to treating the historical circumstances of their emergence as negligible, and *ultimately* to rejecting the notion of progress in philosophy. The second point is made in a brief note co-signed by the two philosophers,(21) also published in 1968, on epistemology in France since 1950: in this tradition, the links between the history of science and the philosophy of science are inseparable. As Granger puts it: "A history of science cannot be reduced to a chronicle of the successive states of science, nor can a philosophy of science be reduced to a structural analysis indifferent to the sequences and twists and turns of its history". Finally, as mentioned above, the traditional idea of a primary philosophy is maintained. Philosophy for the Age of Science is not, as is often the case today, a philosophy of..., of mathematics, of space, of time, of language, of the mind, or of anything else you like, but a philosophy *tout court* in which, of course, mathematics, space, time, language, the mind, and so on. are taken into account. The conjunction of this trait with the first results in philosophical pluralism: the fact that different philosophical systems can coexist is written into the very nature of philosophy. This is arguably a position of common sense, given that the ambition of virtually all the great philosophers of the past to eliminate the doctrines of their predecessors in order to establish their own has been constantly denied by history.

These three characteristics do not exhaust the specificity of this project, but they are enough to distinguish it from traditions outside France as well as from what preceded and followed it in France.

1.0.6. One of the most obvious expressions of philosophical pluralism is precisely the opposition between the doctrines of Granger and Vuillemin. Viewed chronologically, the works of the two philosophers often appear to respond to each other, so that we understand one better when we know the other. Indeed,

(20) This methodology does not introduce an order of dignity between the history of philosophy and the history of ideas; its only aim is to recognise philosophy for what it is.

(21) [Granger and Vuillemin 1968].

with the project of a philosophy for the Age of Science, Granger and Vuillemin circumscribe the limits of a debate in which they occupy two opposing poles. A comparison Granger's Comparative Epistemology and what, for want of a better term, might be called Vuillemin's Comparative Metaphysics highlights the originality of Granger's philosophy, while at the same time setting out the terms of the challenge of his undertaking, which we shall see is so far removed from orthodoxy.

The most striking feature of Comparative Epistemology, when we consider its chronological development, is its constancy, a trait rarely found in even the greatest philosophers. The guiding ideas set out in Granger's first article, published in 1947 in *"Pygmalion": Réflexions sur la pensée formelle* — principally, the proposal to eliminate all forms of dualism — have never been called into question. They have been refined, fleshed out and put to the test, but never abandoned, since this memoir contains the seeds of what would later become the principle of duality. Quite often, we find in the author's earlier articles material for future important works. For example, the book on *philosophical knowledge* published at the end of his career, in 1988, develops the project outlined in the 1959 article "Sur la connaissance en philosophique".

The second feature, far more important than the first, is the abandonment, not to say the devaluation, of metaphysics. This devaluation first manifests as a form of detachment from the history of philosophy. From the very first page of *Essai d'une philosophie du style* Granger tells us that he wants, on the one hand, "to rediscover the truth of Kantianism, by making it completely independent of idealism and a philosophy of consciousness" and, on the other, "to rediscover Aristotelianism as a dynamic philosophy of structures, but freed from its biological paradigms and made independent of an ontology"; reading the great authors of the past having, for him, only the power of suggestion. Then, as we shall see below, by denying any role for metaphysics in the development of scientific works.[22] To which we must add, as we have already said, that there can be no moral philosophy.

Vuillemin, after having long been a Kantian and, as such, believed that philosophy could be a science, became a Platonist. This radical change of position followed — and perhaps even coincided with — the development of a classification of philosophical

[22] Granger does, however, make an exception for Leibniz. We have already noted that he does not hesitate to intervene in the human sciences.

systems. This classification assigns to every philosophy, past, present or future, a place in one of six classes of systems determined prior to the practise of philosophy using a philosophically neutral method.(23) Of these six classes of outdated philosophies, the current state of science (of mathematical logic) leaves only two: realism, of which Plato is the most eminent representative, and intuitionism, of which Kant is the philosophers' favourite example. The great philosophical systems of the past cannot therefore be reduced to a mere suggestive power. Even more so, mixtures such as Aristotelianism and Kantianism are unacceptable, as are all eclecticisms. Faced with the choice between realism and intuitionism, Vuillemin sees in the inadmissibility of intuitionist Moral a sufficient reason to choose realism(24) which attributes to the Philosopher the role of legislator, placing him above all citizens.(25)

At first glance Granger's democratic *aggiornamento* of philosophy — and the dynamism that preserves the possibility of creating new philosophical systems — inspires greater sympathy than the culture of a rigid and antiquated metaphysics that promotes an idea of philosophy that is "perhaps a little too lofty", as Bourdieu noted in his obituary of Vuillemin. But, as Granger tells us, affects must not take precedence over reasons, and we need to understand whether Comparative Epistemology passes the test of the court of science, i.e. whether or not the edifice patiently constructed by Granger accords with the results of science.

§ 2. — The style.

The project of a general stylistics. For Granger, stylistics, as he sees it, is "a new philosophical discipline" in the same way as, for example, the history of philosophy. It sets out to generalise the application of the notion of style, as typically understood in art criticism, to all human productions, whether the most common, such as language, or the most elaborate, such as mathematics. These

(23) Discussed in [Vuillemin 1986].

(24) The choice of realism in Vuillemin's philosophy is not based solely on moral, but also on the philosophy of knowledge.

(25) Our inventory, which is intended only to set the scene, does not exhaust the list of oppositions between the two philosophers. For example, they defend philosophical pluralism on the basis of very different premises.

human productions are the result of a series of acts, i.e. of *work*, the end of which consists in associating a form with a content. It is expressly the study and general analysis of these *processes* that are at the heart of Granger's philosophy, as he already proposed in the first article he published in 1947, and as he explains in the very first lines of the *Essai*: "the relationship between form and content has hardly yet been systematically considered by modern thought as a process, as genesis, that is to say, in short, as *work*". It is therefore the dynamism that presides over human creations that needs to be studied, while remaining faithful to the Kantian heritage, for the undertaking of this general stylistics "should present a certain analogy with that of a transcendental Aesthetic".(26) This analogy should be understood in the light of the fact that "Kantian consciousness gives form, meaning and unity; but [that] it does not work, its activity is graceful".(27) In this way, "the truth of Kantianism" is rediscovered by substituting the acts of men at work for the *ego* in order to constitute a philosophy of practice.. It is then the most general conditions of practice, that is to say, once again, the modalities of work, of the dialectic, which dynamically associates form and content, that will be the object of stylistics. So that "if we were to admit such a big word without smiling" stylistics would be a "transcendental ergology".(28)

In these form-content associations, the emphasis may sometimes be on form, i.e. the abstract, and sometimes on content, i.e. the concrete. The extreme cases of these emphases," says Granger, "appear in the work of the mathematician and that of the labourer. Perhaps we could say that shaping, in the latter case, would consist, for example, in pouring concrete from a wheelbarrow into a bucket; very elementary shaping and work, to be sure, but shaping and work nonetheless.

Granger does not claim to have created a general stylistic. Much remains to be done to put this project to the test. For example, by considering the study of the style of concrete social practices such as political action. However, in attempting to identify the marks of style in scientific thought, and *a fortiori*, in mathematics, he knowingly places himself on what he describes asthe most unfavourable terrain imaginable for testing the possibility of a general stylistics.

(26) [Granger 1968, 11].
(27) *Ibid*.12.
(28) *Ibid. ibidem*. The inverted commas are Granger's.

Definition of style. Granger proposes several definitions of style. The first of these is as follows: style is a way of integrating the individual into a concrete process that is work.(29) This general definition, which is somewhat abstract, could be illuminated by what Granger has to say about scientific progress. The history of science, which is absolutely essential if we are to understand the scope and meaning of current discoveries, "is a *genealogy* of the "categories" that have successively constituted the objects of a science". The sequence of discoveries by creative scientists "ultimately depends only on an *internal movement* of concepts". But "the recognition of such an *internal rationality* in the history of science should in no way detract from the talent or genius of innovative scientists; for it is *individuals* who, by being the first to understand the negative aspects of knowledge already constituted, discover solutions and drive science forward".(30) The individual, in this case the creative scientist, leaves their mark on the concrete process of scientific work. The marks of style are therefore the particular characteristics of the personal contributions of scientists to a discipline in the making. For example, the contributions of Leibniz and Newton to differential calculus have different styles.

A second definition of style, inspired by information theory, describes stylistic facts as the redundant elements, present in individual works, whithin those informational structures that are the constituted scientific theories. It therefore refers to the part of individual work that cannot be assimilated into scientific disciplines or theories as they are anonymously passed down through the legacy of scientific progress.

A function of style. In the second part of the *Essai*, devoted to *style in the construction of the mathematical object*, Granger draws a distinction between stylistic devices that do not affect the structural content of the mathematical concept, which persists despite stylistic effects, and those that give rise to genuine conceptual variations. He illustrates the first case by mentioning the different ways used in mathematics to introduce complex numbers: trigonometric, algebra of square matrices of order 2 and roots of any algebraic equation. The last of these conceptions is undoubtedly more abstract than the first two, but in the end, the operations defined on these different objects will describe the same algebraic structure of a closed commutative field.

(29) *Ibid*, 8.
(30) [Granger 1993, 114-115].

In the second case, on the other hand, to which the book is devoted, "style plays [...] a perhaps essential role, both in a dialectic of the internal development of mathematics, and in that of its relations with more concrete worlds of objects".(31)

In the analyses proposed for this purpose, we can see that Granger, in perfect accord with what we have already seen of his philosophical positions, denies any role for metaphysics, or ontology, in the development of scientific works. This is illustrated in the first study of style — on Euclid — where he proposes(32) to make the Aristotelian doctrine of the incommunicability of genus depend on the style of the mathematicians of his time, rather than the other way round, as scholars of ancient mathematics generally do. For example, for Maurice Caveing, Euclid "skilfully took into account the requirements of the mathematical philosophy of his time" by reconciling the Platonic and Aristotelian conceptions of definition.(33) So much importance is attached to style that it sometimes appears to be no more than a new name for what is usually called "metaphysics" or "theory of knowledge".

2.0.1. The exemplary case of Cartesian style. In this respect, the analysis of Cartesian style is a textbook case in two respects. Firstly, Granger's intention here is "to provide a topical and precise example that can be used to found the very idea of style in this field [that of geometry]".(34). Secondly, because the anchor point for his examination of Cartesian style is the analysis of Cartesian mathematics proposed by Jules Vuillemin, in *Mathématiques et métaphysique chez Descartes*.(35) The title of Vuillemin's work dispenses with any comment what is at stake in the question: does the specific character of Descartes' mathematics stem from a mathematical style or does it depend on metaphysical conditions prior to the practice of mathematics and science in general? There can be no doubt about the nature of this issue. Granger confirms this on the first page of his article on Leibniz. He states that Descartes's philosophical theses, unlike those of Leibniz, do not fulfil "*the function of a conceptual matrix... for the mathematician's constructions*".(36) On the contrary,

(31) [Granger 1968, 21].
(32) *Ibid.* 42.
(33) [Caveing 1990, 130].
(34) [Granger 1968, 43] By comparing Descartes' style with that of Desargues.
(35) [Vuillemin 1960].
(36) [Granger 1994, 199]. Granger's emphasis.

he continues, "the opposite is true. 'The Rules of Method' and the 'order of reasons' have had, if not their source, their guarantee and their model in the shaping of a solution to systems of algebraic equations".(37) In other words, we must consider the cost at which Granger manages to subsume essential aspects of Descartes' treatise under the label of style that we had the best reason to believe depended on metaphysics.

Vuillemin establishes 1°) That the heart of *La géométrie* consists neither in representing curves by equations, nor vice versa, but in showing that curves and equations both depend on the general theory of proportions. In other words, the only mathematical entities admissible in *La géométrie* are those constructible by the five rational operations which can be defined using the theory of proportions.(38) 2°) In direct relation to the theory of proportions, Descartes classifies curves according to their *genus*, i.e. according to the number of proportions needed to construct them. The concept of genus, which does not correspond to that of the degree of an equation, can, or should, be seen as an analogue of the contemporary concept of measuring the complexity of a demonstration.(39) (40) 3°) Correlatively, Descartes deliberately eliminated transcendental curves from *La géométrie*, even though he knew all the properties of those he occasionally encountered, such as the equiangular spiral and logarithmic curves. The curves admissible in geometry must be constructible point by point; they cannot be generated by two independent movements; their analysis cannot depend on infinite reasoning.

Vuillemin shows that these drastic constraints that Descartes imposes on *La géométrie* are dictated by metaphysical considerations: "the divine incomprehensibility, which serves as a correlate to the irreducibility of sensation (*sentiment*), plays the same role in

(37) *Ibid. ibidem*.

(38) The whole of *La géométrie* follows from the theorems of Thales and Pythagoras, wrote Descartes to Elisabeth.

(39) See the table of correspondences between the number of lines in the Pappus problem, the genus of curves and the degree of the equations in [Vuillemin 1960, 109]. Incidentally, we can see how arbitrary it is to count *La géométrie* as a chapter of analytic geometry. In his annotated translation of [Smith and Latham 1925], D.E. Smith dismisses the concept of *genus* out of hand, translating it as *class* and adding that today *genus* means something else.

(40) A more precise presentation of the analogy between the Cartesian method and the current concept of the complexity of a demonstration can be found in our forthcoming study on *The Cartesian concept of function*.

Metaphysics as we would see it play in *La géométrie*, if we were to delve deeper into the foundations of the latter science, where alone it can also take charge of the transcendence of mechanical curves".[41]

2.0.2. The autonomy of mathematics and the dismemberment of Cartesian thought. Granger departs from Vuillemin's analyses on two main points. Firstly, he completes them by reminding us that for Descartes mathematics must be applicable and that he has a distaste for pure mathematics.[42] Above all, he observes that *La géométrie* is a metric geometry. Its sole purpose is to measure the length of line segments. Secondly, he refused to give the theory of proportions, and the sliding squares introduced in Books II and III of *La géométrie* as a machine for calculating proportions, the central roles attributed to them by Vuillemin.

On the basis of these considerations, Granger set out to show that the two styles of the science of extent, the Cartesian metric style and the projective Arguesian style, would cease to be styles when Klein combined the points of view of Descartes and Desargues with the theory of transformation groups.[43] In the case of Descartes, who introduced the concept of algebraic structure *in nuce*, this stage was preceded by Abel and Galois clarifying those structurals aspects that had eluded him.[44]

To justify what must be called a teleological conception of history[45], Granger presents *La géométrie* "a science of extent" — an expression he uses six times — without giving, and for good reason, any text to support such an interpretation. This idea is belied by the very fact of the metrical nature of the Cartesian treatise. In truth, this metrical character and the demand for the applicability of mathematics are one and the same. In fact, *La géométrie* is essentially a reworking of the "general science that explains everything that can be researched concerning order and measurement, without assigning it to any particular subject" that Descartes conceived in the *Rules for the Direction of the Mind*. If *La geometry* truly were a science of extent, it would be difficult to account for the fact that Book II is largely devoted to optics, or why Descartes devotes

(41) [Vuillemin 1960, 97].

(42) It is well known that Descartes' papers contained a proof of Euler's relation for convex polyhedral [Descartes 1987], which he never made known.

(43) [Granger 1968, 46].

(44) *Ibid*, 54.

(45) This criticism has already been made by [Michel 2000], as noted by [Crocco 2024].

so much of it to supersolids problems. If, as Vuillemin observes, Descartes comes very close to discovering the concept of dimension — namely, the physical concept of dimension, in the sense of dimensional equations. The apparent ambiguity of this notion, which might lead one to think that it could refer to the dimensions of space, can be explained by the fact that Cartesian mechanics is cinematographic and reduces all its quantities to instantaneous descriptions of products of lengths, as Martial Gueroult has compellingly demonstrated.(46)

As for the accessory role Granger gives to the theory of proportions and the dominant role he gives to the form of a solution to systems of algebraic equations and to the classification of algebraic curves, they directly contradict Descartes' stated intentions. In the *Second Part of* the *Discours*, which is an introduction to *La géométrie*, Descartes tells us that his intention was not

> "to learn all these particular sciences, which are commonly called mathematics,(47) and seeing that even though their objects are different, they all agree in that they do not consider anything other than the various ratios or proportions found in them, I thought it would be better for me to examine only these proportions in general".

He later adds that he intende to apply this method just as effectively to the difficulties of the other sciences as he had done to those of algebra.(48) This promise was fulfilled both in *La dioptrique* and, again, in Book II of *La géométrie*.

By isolating *La géométrie* from what conditions it, namely metaphysics — which certainly cannot be reduced to the Rules of Method — and from what it conditions, namely the possibility of mathematical physics, and by unjustifiably lending it the role of a science of space, Granger embeds *La géométrie* in an illusory history of mathematics from which Descartes excludes himself. Only through this surreptitious dismemberment and misappropriation of Cartesian thought does it become possible to speak of a Cartesian style in geometry.

(46) [Gueroult 1956, 276]

(47) These are, of course, the four liberal arts (arithmetic, astronomy, geometry and harmonics), to which, at least, algebra, mechanics and optics must obviously be added.

(48) [Adam and Tannery, 19-21].

§ 3. — The principle of duality.

We saw (in 2.1) that style characterised what was individual in the works of scientists and that, as such, it was opposed to the structures of science understood as constituted and transmissible knowledge. What individual works and established science share is a common foundation :the principle of the duality of operation and object. But whereas in scientific works this principle is associated with elements that are foreign to it, in constituted science it is in all purity, so to speak, stripped of all over-determination, that this principle presents itself.

Nature of the principle of duality. Granger defines the principle of duality as *the principle of the necessity of a reciprocal determination of any system of objects of thought and an associated system of intellectual operations.* This principle is a "primitive category of thought, insofar as thought is knowledge of objects". By *category* Granger means the ultimate condition of an act of knowledge whose operation is independent of a division of the knowable into regions of objectification. The category of duality must therefore not be confused with the categories specific to scientific disciplines, as expressed in their specialised languages (cf. 1.2.1).

The principle of duality, which is not a mathematical principle, since it operates whithin all knowledge of objects, is nevertheless inspired by mathematics, notably through two features: the reciprocity of points of view and the permutation of objects and operations. The first feature is exemplified by the pair of Pascal's and Brianchon's theorems — and, we might add, by all the theorems of projective geometry, which always go in pairs: in the first case, we take the point of view of the points and, in the second, the point of view of the lines. The second feature is illustrated by the concept of a dual vector space. The linear forms — which are operations — that apply to the vectors of a given space E — which are objects — into the field of E^* are the vectors (objects) of the dual space E^*.

We saw (2.1) that Granger's main idea was to consider the opposition between form and content as a process. This thesis is justified by at least two facts. On the one hand, in the sciences we never encounter absolute forms or contents. The opposition is always relative. Secondly, the same applies in natural languages, where the opposition between syntax and semantics is likewise relative:

what is "syntactic" at a certain level of analysis becomes "semantic" at another level.(49) Syntactic forms are not passive receptacles of content; they themselves have content.

Although the principle of duality is at work in all the sciences, a distinction must be made between the empirical sciences, which maintain links with perception via observations,(50) and those in which such links are non-existent.

Duality in the empirical sciences. Duality functions in the empirical sciences to the extent that they have succeeded in constituting their objects. To illustrate this, Granger cites the example of the concept of temperature, which can appear either as content or as form.

In the law of gas expansion (Charles' law) :

$$PV = NkT$$

temperature T is an empirical datum functioning as a parameter within a structure; the concept of temperature functions as a content. On the other hand, in the kinetic theory of gases, where the concept is defined as the average kinetic energy of a set of molecules, temperature is a form, or a structure, which acts as an operation.

This immediately raises the question of how duality might pperate in disciplines such as ibiology, or even chemistry.(51) Granger's principal interests have consistently centered on mathematics, linguistics and the human sciences. It is therefore hardly surprising that he does not mention biology. Personally, we only know of one text in which there are brief allusions to it.(52) One, which seems positive, refers to "the very recent idea of a computerised genome",(53) the other, frankly negative, contrasts the clarity, coherence and precision of Maxwell's electro-magnetic theory with

(49) For example, the morphological level is semantic in relation to the syntactic division into phonemes, but it is syntactic in relation to grammatical analysis.

(50) "Observations" should be taken in the broadest sense of the term. It can mean observing a phenomenon or reading a measurement on an instrument.

(51) In his analyses of the relationship between the history of science and history, epistemology and technology — an area in which his theses are extremely convincing — Granger always seems to confine himself to physics.

(52) The 1985 edition of "Événement et structure en histoire des sciences" is reprinted as chapter nineteen of [Granger 1994]. But, let us repeat, we certainly do not claim to be an exegete of Granger's thought. These allusions to biology can perhaps be explained by the fact that here the author is partly discussing the theses of G. Canguilhem.

(53) [Granger 1994, 366]

that of most biological theories which "are much less close to this ideal".(54) In truth, Granger shares Kant's view that "The theory of nature contains science properly so called (pure) only in so far as it contains mathematics".(55) If Ganger remains silent on biology, it is perhaps not because mathematics, language and the human sciences are enough for him, but rather, it seems, because, as this science is not formal, it does not deserve his attention.

The concept of formal content. When the principle of duality is applied to a material that has nothing to do with empirie, it produces *formal content*. The production of formal contents appears in three domains: language, mathematics and, to a certain extent and in a limited form, logic. In truth, the concept of formal content fulfils three main functions. First, it provides a criterion for demarcating logic from mathematics. Secondly, it provides a substitute for the existence postulates in mathematics by saving the spirit of Hilbertian formalism.(56) Finally, it makes it possible to formulate a hypothesis about the universals of language, thus answering the problem mentioned above (1.2.3).

Here again, the analytical links between these three functions are based on a recasting of the Kantian heritage.

In classical philosophy, the opposition between form and content was considered to be either ontological or epistemological. Granger adopts what he calls a different, "more radical" hypothesis, he says: "form and content cannot be prior to a symbolic representation of experience. Signifying *and opposing form to content are two correlative, indeed inseparable, operations*.(57) It might have been more appropriate to call this hypothesis, inspired by a reflection on the nature of language and mathematics, a "thesis", since it echoes the remodelling of Kantianism already undertaken with the idea of a "transcendental ergology" and the elimination of the passivity of the mind. With formal content

(54) *Ibid*. 369. When Granger was asked, more than half a century ago, about his lack of interest in biology, he replied that we did not know how to cure cancer. The sad thing is that the future has continued to prove him right.

(55) [Granger 1960, 11].

(56) "The mathematician's actual work is hardly conceivable in [a] pure universe of signs...At each stage, he aims at a universe of objects whose *existence* is undoubtedly an *operative* necessity, but from which his Great Work paradoxically consists in freeing himself, once the whole edifice has been erected". [Granger 1979, 59].

(57) [Granger 1994, 34]. Emphasis added.

> "The question of the analytic and the synthetic in scientific knowledge is taken up again in one of its aspects. If it is possible to characterise certain aspects of this knowledge [i.e. mathematical knowledge] as formal, and at the same time to recognise their content, we will perhaps have effectively displaced the problem posed by the fruitfulness of supposedly analytic knowledge, and substituted for the idea of synthetic *a priori* that of formal content".(58)

In other words, by remaining faithful to the spirit of logical positivism and Hilbertian formalism, we can avoid the ruinous thesis that mathematics is nothing but a series of tautologies. Now, since the opposition between form and content is relative, and since to signify is to oppose a form to a content, we must reject the idea of the forms of sensible intuition as definitive and *a priori* frameworks(59) and reinterpret Transcendental Aesthetics as Transcendental *Semiotics*.(60)

The concept of formal content would thus make it possible to explain the nature and fruitfulness of mathematical knowledge by adopting a form of constructivism that rejects the "psychologism" of intuitionism and rules out the idea that it can be reduced, as Hilbert claimed, to a pure universe of signs.(61)

Formal content arises when operations are imperfectly matched with the objects to which they are applied. They are then opaque.

(58) [Granger 1994 33].

(59) For Granger, the study of the possible existence and nature of forms of sensitive intuition is a matter for psychology or, perhaps, neurology [Granger 1994, 35].

(60) [*Ibid*.] Does Transcendental Semiotics coexist with Transcendental Ergology or does it replace it? This is a question that we are unable to answer, probably because we do not have a complete grasp of Granger's thought. The fact remains that Transcendental Ergology was presented as an analogue of *Aesthetics*, whereas Transcendental Semiotics is supposed to be its substitute. Since, moreover, transcendental Semiotics occupies a more fundamental position, since it is supposed to be the repository of universal proto-logical conditions of linguistic expression, it is safe to assume that the postulate of a transcendental Ergology disappeared in this later stage of Granger's thought.

(61) The idea that Hilbert wanted to reduce mathematical knowledge to a pure universe of signs often provokes cries of protest. Granger did not fall into this trap. It has to be said that the texts are stubborn: "Am Anfang ist der Zeichen", says Hilbert to oppose his point of view to those of Brouwer, Cantor, Dedekind, Frege, Kronecker, Poincaré and Weyl. The fact that this purely semiological doctrine contradicts Hilbert's own mathematical practice only confirms its inadequacy.

But the exercise of operations, in a way illegitimate, "eventually leads to a restructuring and extension of the primitive domain, restoring, at least provisionally[62] the transparency of the object".[63] The well-known example of the irreducible case of the cubic equation, encountered by sixteenth-century mathematicians, provides a simple illustration of this situation. When we apply the usual algebraic rules to find the real roots of the equation, whose existence we are sure of, we come up against an impossible entity in the extraction of the square root of a negative number. Despite this obstacle, mathematicians accepted this operative extension without justifying it for more than two centuries, until a geometric representation of the imaginary "finally gave them a suitable status, re-establishing a non-contradictory correlation between the system of operations of algebra and the system of objects that these operations determine and manipulate.[64]

Logic and formal content. Logic, the most obvious source of purely formal thought, is a borderline case as far as the production of formal content is concerned. "The form-content relationship is presented there at its *zero degree*, the object being only the support *without qualities* of the system of operations that determines it",[65] as we see with the classical calculus of propositions. This is true whether we interpret the propositional object as a "utterance" of a language or as a "class". "Here the dualitý operation-object is perfect: there is nothing *opaque* [emphasis added] in this object whose knowledge is exhausted in that of the operations it supports, as a simple *presence or absence* [emphasis added]".[66] At this formal level, the perfect correlation of operation and object is manifested in the meta-properties of propositional calculus: non-contradiction, completeness, decidability. In contrast, the predicate calculus shows that the distinction between the objects "individual"

[62] Until further notice because, by definition, science is unstable and the structure updated by the adjustment of objects to the rules is bound to be integrated in the future into a more general and abstract structure.

[63] [Granger 1994, 65].

[64] *Ibid. ibidem*. The example proposed by Granger does not seem completely convincing insofar as it is supposed to illustrate the fact that "the objectal overflows the operative" whereas it seems rather to present a deficiency of the object in the face of the rules.

[65] [Granger 1994, 61].

[66] *Ibid. ibidem*.

and "property" attenuates the transparency of the object by depriving the calculus of decidability. Logic in the strict sense is therefore that domain of formal thought where the application of the category of duality results in a complete adequation of the operational (*opératoire*) and the objective (*objectal*), so that no formal content appears.(67) Logic *stricto sensu* then merges with *analytics*.(68)

Deviant calculi, such as modal logics, also involve coarse forms of objects. What's more, the symbolic manipulations in these non-classical calculi are carried out in accordance with the rules of the propositional calculus, since every statement "is in fact *posited or non-posited* (*posé ou non-posé*) [emphasis added]. The *classical calculus therefore acts* as a universal *meta-system* for the most aberrant systems".(69)

Intuitionistic logic deserves a special place, as its relationship to classical logic is not one of mere subordination. Trivially speaking, there are theorems in classical logic that are not intuitionistic theorems. On the other hand, via a certain translation, we can show that a proposition is valid in intuitionistic logic if and only if it is classically valid. But there are theorems in intuitionistic logic that are not the translation of any classical theorem.(70) But, adds Granger, it's only when we consider their application in universes of objects where we can distinguish finite collections from infinite collections that the two logics differ significantly. Consequently, with the introduction of infinite collections, we enter the realm of mathematics, where the articulation between the operative and the object (*objectal*) becomes more complicated, thereby generating some formal content.

§ 4. — Diagnostic elements.

Does this conception of logic, which is reminiscent of Quine's in many respects, fit in with the current state of mathematical logic, which is a science whose object is the study of correct reasoning? To carry out this examination, let us follow Granger's presentation by considering first the question of the nature of the calculus of propositions and, secondly, that of the unicity of logic.

(67) *Ibid. ibidem*.

(68) [Granger 1994, 62].

(69) [Granger 1994, 63].

(70) Granger relies here on [Scott, 1981]. Unfortunately we were unable to consult this reference, and in what follows we give credence to Granger's rendering of it. This point is not insignificant for the discussion below (4.2).

Properties of the classical calculus of propositions. In the first place, the gloss that comments on the calculation of propositions is quite obscure. The meaning of *not A* returns to the absence of *A* or to the case where *A* is not posited. How then should *not not A* be interpreted? As the absence of an absence? As the non-posedness of what has not been posed? It is preferable to set aside these considerations, which stem from the rather unfortunate influence of the Wittgenstein of the *Tractacus*, while keeping in mind that they concern the nature of negation, and instead focus on the technical considerations invoked by Granger. In this respect, the characterisation of the calculus of propositions by its "meta-properties" is hardly any happier. First of all, when a system is decidable, it is superfluous to invoke its completeness, which is only useful when a system has no decision procedure. Secondly, this characterisation does not single out the calculus of propositions, as Granger points out when he reminds us that there are consistent, decidable and complete modal logics. To claim to eliminate these obvious counterexamples on the pretext that they contain "coarse forms of objects" is a pure statement of principle, because either the proposed criterion is discriminating or it is not. What really sets the calculus of propositions apart is its functional completeness. But this property is based on the fact that it essentially has only one connector, as Sheffer has shown, whereas other logical systems have at least two. One is unary, for negation, and the other is binary, equivalent to an analogue of the classical conditional. Negation and the other connectors of classical calculus are therefore merely convenient means of expression, because they are closer to our linguistic usage. Curiously, textbooks on elementary logic confine Sheffer's connectors to the role of calculation exercises for beginners, neglecting the fact that they tell us something profound about the nature of propositional calculus, namely that it has no independent negation as such or, correlatively, that the nature of its conditional seems, if not artificial, at least inappropriate to represent the operation "if... then..." as it is used in mathematical reasoning.[71]

Secondly, despite its simplicity, does the calculus of the propositions present the transparency that Granger sees in it?

To the obscurity of the opposition between the posited and the non-posited, Granger, believing himself to be defending a form of

[71] This is the starting point of [Anderson and Belnap 1975]'s work on the logic of relevance, which aims to reconstruct real mathematical reasoning in which the antecedent of a conditional must always be related to the consequent.

generality, adds confusion when he asserts that the object "proposition" can designate either a utterance or a class, in other words a material entity (utterance) or an immaterial entity (class). Let us mention two well-known facts that cast the heaviest doubts on this supposed transparency of the calculus of propositions.

Firstly, the general question of simplifying propositional calculus formulae — i.e. finding the simplest possible equivalent formula for a given formula — which has mobilised generations of engineers. As Quine points out, it is remarkable that no general and rapid method is known for reducing a formula in disjunctive normal form to one of its briefest equivalents.(72) The author confides that in 1948, when he was preparing the first edition of *Methods of Logic*, he had hoped to base his entire logical treatment of truth functions "on a method of mechanical and easy simplification", without then having realised the elusive nature of such a technique.(73)

The problem addressed by Quine primarily concerned the design of electrical circuits. The second fact that should be mentioned is more theoretical in nature, even if it is permissible to wonder whether it is not fundamentally a similar problem with a very different aspect. It concerns the famous enigma of theoretical computer science: *P=NP?* and proof that the decision problem of propositional calculus is NP-complete.(74) Under these conditions, it seems difficult to assert that in the calculus of propositions "the operative completely dominates the objectal and the objectal only reflects the operative; no property of the "object without quality" that is not then completely reducible to the operative, i.e. to a finite and explicit demonstration, according to an effective canonical procedure".(75)

Classical logic and deviant logic. The old adage *traduttore, traditore* would not be true if there were only one, and only one, way of translating the logical constants of classical language into those of intuitionistic language, or vice versa. Since, in fact, several different translations are known,(76) it is preferable, if not essential, to

(72) [Quine 1972, ch. 11]. See chapter 11 of this work for a discussion of this question, which we can only allude to. A reconstruction of Quine's presentation that is fairly consistent with our intentions can be found in [Feghaly 2016, 166-167].

(73) [Quine 1972, Historical note of ch.11].

(74) For a presentation [Hopcroft and Ullman 1979].

(75) [Granger 1994, 61]

(76) At least the well-known one by Gödel and the one by Scott mentioned in footnote 70 above.

use another method to compare classical and intuitionistic logics in an intelligible way and to understand what really distinguishes them; this without postulating that the logical constants of one of them must be considered as norms to which the logical constants of the other are subject.

Gentzen designed the sequent calculus precisely to deal with these questions, putting intuitionistic logic and classical logic on the same footing.

As we know, there are two types of rule in the sequent calculus: structural rules and logical rules. Structural rules describe the properties of objects, regardless of their nature, to which logical rules apply; they express the conditions under which an object may be swapped, deleted, duplicated and added.(77) The rules governing the introduction and elimination of connectors are common to both logics. The differences between classical and intuitionistic connectors are the consequence of the fact that the right-addition rule

$$\frac{X \vdash Y}{X \vdash Y, B}$$

is not allowed for intuitionistic logic. This condition amounts to admitting only one conclusion, and one only, to the right of the deduction sign $\vdash$, and therefore, as a very simple reasoning shows, to "blocking" the formulation of the principle of the excluded middle.

The sequent calculus is therefore a philosophically and linguistically neutral framework that allows us to clearly and distinctly understand the differences between classical logic and intuitionistic logic. But it is much more than that. As Dana Scott points out, it should primarly be regard as a framework for representing the "most general principles of deductive inference" rather than as a tool of Proof theory.(78) In this spirit, Gentzen's ideas have been generalised by his successors in various directions. Two of these are relevant to our subject.

4.0.1. Logic of any object. By enriching the structural rules of the sequence calculus, Kosta Došen has proposed an answer to the fundamental question "What is a logical constant?"

If we accept, in accordance with Gentzen's intentions, that logic is the science of formal deductions, then we can consider that the

(77) With the rules of exchange, contraction and weakening, duplication being a special case of weakening.

(78) [Scott 1971, 793].

basic deductions of logic are *structural deductions*. In other words, deductions in which only structural rules appear. The logical constants that enable us to formulate the premises and conclusions of deductions in object languages are therefore absent from structural deductions, so that the latter are independent of the former. Any logical constant present in a deduction can ultimately be analysed in structural terms. At least until proven otherwise, this result having been established for the usual constants, including quantifiers and equality. Logical form therefore manifests itself essentially in structural deductions. In this view, constants can be seen as punctuation marks in the object language for certain structural features of deductions.(79)

The characterisation of the necessity operator in modal logic deserves a moment's thought. Usually A, B, ... and X, Y, ... designate respectively logical formulas and lists of logical formulas in the object language. But, as we have said, we must not prejudge the nature of the objects in the structural rules, since this is determined by the rules themselves. So we can consider that A, B, ... and X, Y, ... no longer designate object language formulas but sequents. By adding structural rules for sequents of sequents, Došen reduces the modalities to structural deductions, as he does for the other constants. The "deviant logics" do not therefore contain, as Granger claims, "coarse forms of objects" but elaborate forms of deduction that strictly exploit the principle of duality by making the original operations play the role of objects.

Since logical deductions are independent of logical rules, which are identical for the different logical systems, it can be said that logic is independent of the domains for which object languages are designed, as Gentzen established for classical and intuitionist logics. This conception of logic can therefore be said to be that of any object.

4.0.2. Logics of particular objects. The previous considerations were developed by enriching the structural rules. Symmetrically, we can draw on the resources of the sequence calculus by enriching the logical rules and suppressing some structural rules. The latter approach allows for a more precise meaning for the otherwise vague notion of posed and non-posed by distinguishing different domains of logic.

(79) [Došen 1989].

The justification of mathematical reasoning has been the main, if not the only, source of the origins and developments of contemporary logic. This is particularly true of Proof theory, often referred to as metamathematics. However, when we consider using logic to design programming languages or to develop methods for verifying that a programme terminates, can we accept that the objects linked by the logical constants of the object language can be of the same nature as mathematical propositions? It was by analysing this question that Jean-Yves Girard devised linear logic.[(80)]

Rather than summarising Girard's analyses, we may briefly capture their spirit by using a Chinese proverb:[(81)] "If we each have an egg and exchange it, we will each have an egg. If we each have an idea and we exchange it, we will each have two ideas". It should be added that once an egg has been consumed, it can no longer be used, whereas ideas do not wear out, even when they are used. Mathematical propositions can be likened to ideas, while the resources of a programme resemble eggs. This distinction clears up the confusion caused by Granger's ambiguous use of the French verb "poser", which has a concrete meaning — like I put my glass on the table — and an abstract meaning — like I pose this hypothesis. *To pose* is opposed to to remove (*enlever*) for entities with material features similar to those of eggs, while the opposition of *true* to *false* is appropriate for mathematical propositions.[(82)]

One of the key ideas of Girard's linear logic, from the point of view we are interested in, can be formulated as follows: is there a logical system which a) shares the constructive nature of intuitionism, b) whose logical constants (and in particular negation) verify laws analogous to those of De Morgan, and c) which takes into account the possibility that A, B,... and X, Y,... might denote material entities? The positive answer is achieved by eliminating all the structural rules of the sequence calculus with the exception of the right permutation rule and the cut rule. The logical constants thus obtained are more numerous than those of classical, intuitionistic and modal logic. The logical rules associated with them are context-dependent. For example, there are two conjunctions, multiplicative and additive,

(80) See "Logique et informatique. Le point de vue d'un logicien" in [Girard 1988].

(81) Readers are still advised to consult the reference in the previous note for a clearer idea of the distinction between mathematical objects and computer objects.

(82) With the reservation that it only concerns the denotation of these propositions; $2+2=4$ and $3\times 3=9$ have the same denotation (*true*) but obviously do not have the same meaning.

depending on whether the contexts of the formulas are different or identical

$$\frac{\vdash A,C \qquad \vdash B, D}{\vdash A\wedge B, C, D}\,\mathrm{m} \qquad \frac{\vdash A, C \qquad \vdash B, C}{\vdash A\wedge B,\ C}\,\mathrm{a}$$

In the same way, two disjunctions are distinguished because there is total symmetry between the logical constants in linear logic.(83) Girard points out that the multiplicative disjunction is the least apparent constant in linear logic. It is revealed in a purely formal way as the dual of the additive conjunction.(84)

In addition to showing that there is no generic notion of the posed and the non-posed, these considerations also show that the spontaneous, or "intuitive", conception of logical constants associated with classical propositional calculus is only a residue of a pre-scientific vision of logic.

§ 5. — **Conclusions.**

Granger explicitly rejects any analogy between geometric pluralism and logical pluralism because Euclidean geometry, he says, "is not a universal meta-system for other geometries, which [...] can be developed independently [...] *without its operative help*".(85) Whereas, by virtue of the doctrine of the posited and the non-posited, "*classical calculus plays the role of* universal *meta-system* for the most aberrant systems".(86)

Whatever Granger thinks, the parallelism between geometry and logic seems perfectly justified. The sequent calculus has played the same role in relation to logical pluralism as group theory has in relation to the problem of space. It even offers a striking example of the effectiveness of the principle of the duality between operation and object in scientific methodology.

There is no common core of logic in the sense that it contains logical rules for any objects. This notion of "the logical" is merely a view of the mind, an illusion of pure reason. In any logical system, the logical rules are affected by the nature of the objects, described by the structural rules, on which they operate, and vice versa.

(83) A concise and very accessible presentation of the constants of linear logic, and of negation in particular, can be found in [Mélès 2009].

(84) [Girard 1987, 5].

(85) [Granger 1994, 63]. Granger's emphasis.

(86) *Ibid.ibidem*. Granger's emphasis.

The sequents calculus gives full expression to the idea of a philosophically neutral unity of logic. As such, it is opposed to all doctrines that, like Granger's, advocate the unicity of logic. In other words, contrary to what Granger believes, logical negation is not the product of a primitive category of thought: it is a concept.

The solution to the problem of the unity of logic does, however, vindicate Granger's doctrine on one point: logic is independent of mathematics.

Given that logic is independent of mathematics and that duality underpins its philosophical neutrality, might we not amend Granger's philosophy by depriving it of its retrograde and illusory doctrine of logic? Let us first note that if Granger had not pronounced himself on the nature of logic, his philosophy would have been incomplete. It would therefore be necessary to "rectify" his doctrine by choosing a logic and resolving to let in through the window what style had driven out through the door, namely the compromise between mathematics and either the "psychologism" of intuitionism or the Platonic realism of forms. Such an outcome would amount to the ruin of his system, which was built precisely to avoid this alternative. Duality, the keystone of Granger's system, therefore fails to live up to its promise, since it cannot account for what *justifies* mathematical knowledge. Conceived as a primitive category of thought, it is no more than a principle of scientific methodology, admittedly an important one, but one that is only effective in certain areas of scientific knowledge. For example, this principle appears unable to account for the concept of real numbers and the distinction between differentiability and continuity, and therefore for the foundations of elementary analysis.[(87)]

Is a stylistics orphaned by duality conceivable? We would be confusing the order of construction of Granger's philosophy, in which stylistics is chronologically promoted before duality,[(88)] with his order of reasons, in which duality commands stylistics. Deprived of its horizon, duality and nothing but duality, stylistics is reduced to a set of isolates that could just as easily be called "a few elements of the history of mathematics".[(89)]

[(87)] For both depend on a position on the nature of infinity. On this point see Gentzen's article "The concept of infinity in Mathematics" in [Szabo 1969].

[(88)] With, however, the reservations expressed above in the second paragraph of 1.3.3.

[(89)] With "elements" in the usual sense of the term, and not in Euclid's sense, and with a history of mathematics suffering from "mathematism", as we saw above (2.3.2).

For a follower of Philosophy for the Age of Science, this failure has an immediate and important consequence. Bringing philosophy truly down to earth, to the level of working men and women, is a highly appealing programme. Before making the admirable effort to bring it to fruition, Granger proposed that we should resolutely turn our backs on old philosophical questions such as "What is there?" The failure of his doctrine brings us back to this point, at least temporarily. Fortunately, this has been done without having to abandon the precepts of Philosophy for the Age of Science, and by drawing the greatest possible benefit from a form of concordance between logical pluralism and philosophical pluralism as conceived by Comparative Metaphysics. Thus, after recalling that the concepts of formal content and duality are the most difficult to analyse, because "it is from their opacity that their fruitfulness is born", Vuillemin rightly urges Granger to choose between the realist, intuitionist and naturalist conceptions of logic.(90)

Followers of Philosophy for the Age of Science are not the only ones concerned with the relationship between philosophy and mathematics. We might then ask whether, from this examination of Granger's intentions, we could not draw a general lesson that would apply, for example, to the entire readership of the *Annals for Mathematics and Philosophy*. Perhaps this would include those who feel that the notion of a philosophical system is nothing more than an abstruse and cumbersome burden when it comes to understanding the real workings of mathematical creation and its methods of justification. We could simply abandon Granger and, armed with a new notion of style,(91) study the works of mathematicians past and present to reveal their "philosophical ideas". But when we came to study the work of Gauss, the Prince of mathematicians, we would come across his well-known dictum: the use of an infinite quantity as a complete thing is never admissible in mathematics.

Mathematicians work with a given material. At the very least, this material contains the natural integers. *All* the integers or the *whole* of integers? This question of existence is unavoidable. Contrary to what some people claim, it cannot be reduced to the syntax of the symbol $\exists$. Taking it into account certainly does not exhaust the analysis of the works of mathematicians. To ignore it, to ignore the B, A, BA of the philosophy of mathematics, is to risk going astray in the attempt to philosophise.

(90) [Vuillemin 1987]

(91) The marketplace of ideas is not short of them, as the article *Style* in the *Stanford Encyclopedia of Philosophy* shows.

§ — References.

Adam et Tannery. 1996. Charles Adam et Paul Tannery, *Œuvres de Descartes, vol.VI*, Paris, Vrin, 1996.

Anderson et Belnap. Alan Ross Anderson and Nuel D. Belnap, *Entailment, The Logic of Relevance and Necessity*, Princeton et Londres, Princeton University Press, 1975.

Caveing 1990. Maurice Caveing, *Introduction* in *Euclide, Les éléments*, vol.1, trad. et commentaires par Bernard Vitrac, Paris, PUF, 1990.

Costa 2013. Andrea Costa, « *Parco cæteram Usinulcæ amentiam memorare*. Approches de la stylistique leibnizienne : l'*Iconologia* de Cesare Ripa et les *exempla* romanesques », *Lexicon Philosophicum: International Journal for the History of Texts and Ideas*, n° 1, 2013.

Crocco 2024. Gabriella Crocco, « Mathématiques et profondeur : la philosophie mathématique et le problème de l'histoire », https://www.college-de-france.fr/fr/agenda/seminaire/la-philosophie-de-la-pratique-des-mathematiques/mathematiques-et-profondeur.

Descartes 1987. René Descartes, *Exercices pour les éléments des solides*, Édition critique avec introduction, traduction, notes et commentaires par Pierre Costabel, Paris, PUF, 1987.

Došen 1989. Kosta Došen, « Logical Constants as Punctuation Marks », *Notre Dame Journal of Formal Logic,* vol.10, Number 3, 1989, p.362-381.

Feghaly 2016 « Les frontières entre la logique et les mathématiques : le point de vue de Gilles-Gaston Granger », *Philosophia Scientiae*, 20-2, 2016, p.159-175.

Girard 1988 Jean-Yves Girard, « Philosophie de la logique linéaire : logique et informatique, point de vue d'un logicien ; la logique comme science de l'interaction », 1988, URL : http://unesdoc.unesco.org/images/0008/000832/083213fb.pdf.

Girard 1987 « Linear logic », *Theoretical Computer Science*, vol.50, 1987, p.1-102.

Granger 1960. Gilles-Gaston Granger, *Pensée formelle et sciences de l'homme*, Paris, Aubier 1960 ; Sec. éd., 1967, avec l'adjonction d'*Adresse aux lecteurs sur le structuralisme*.

Granger 1968. Gilles-Gaston Granger, *Essai d'une philosophie du style*, Paris, Armand Colin, 1968, sec. Ed. revue et corrigée, Paris, Odile Jacob, 1988.

Granger 1979. Gilles-Gaston Granger, *Langages et épistémologie*, Paris Klincksieck, 1979.

Granger 1985. Gilles-Gaston Granger, « Pour une épistémologie du travail scientifique », *La vie des sciences*, Comp*tes rendus de l'Académie des Sciences*, série générale, tome 2 n° 4, p.367-374, 1985.

Granger 1987. Gilles-Gaston Granger, *Leçon inaugurale au Collège de France*, Paris, Éd. Du Collège de France, 1987. Rééd. In *Noesis*, **38**, 2022, p.251-269, https://journals.openedition.org/noesis/7423.

Granger 1988. Gilles-Gaston Granger, *Pour la connaissance philosophique*, Paris, Odile Jacob, 1988.

Granger 1993. Gilles-Gaston Granger, *La science et les sciences*, Paris, PUF, 1993.

Granger 1994. Gilles-Gaston Granger, *Formes, opérations, objets*, Paris, Vrin, 1990.

Granger 2003. Gilles-Gaston Granger, *Philosophie, langage, science*, Paris, Edp Sciences, 2003.

Granger et Vuillemin 1968. Gilles-Gaston Granger et Jules Vuillemin, « Tendances de la philosophie des sciences en France depuis 1950 », *Contemporary Philosophy. A Survey. – La philosophie contemporaine. Chroniques*, ed. Raymond Klibansky, La Nuova Italia, Firenze, 1968, p.161-163.

Gueroult 1956. Martial Gueroult, *Descartes selon l'ordre des raisons, vol. 1*, Paris, Aubier, 1956.

Hopcroft et Ullman 1979. John Hopcroft et Jeffrey Ullman, *Introduction to Automata Theory, Languages and Computation*, Addison-Wesley, 1979.

Mélès 2009. Baptiste Mélès, *La négation en théorie de la démonstration, de Gerhard Gentzen à Jean-Yves Girard*, hal-01225141, 2009.

Michel 2000. Alain Michel, « Mathématique et « profondeur » : l'exemple de la théorie des nombres », in G. Ravis-Giordani, *Jean Toussant Desanti, une pensée et son site*, Paris, Editions CNRS 2000, p.181-199.

Scott 1971. Dana Scott, "On engendering an illusion of understanding", *Jour. of Philosophy*, 1971, p.787-807.

Scott 1981. Dana Scott *et al.*, *Notes on the Formalization of Logic*, Oxford, Oxford University Press, 1981.

Smith and Latham. David E. Smith and Marcia L. Latham, *The Geometry of René Descartes*, La Salle, Open Court, 1925. Reprint, Dover Books, 1954.

Szabo 1969. *The Collected Papers of Gerhard Gentzen*, ed. By M. E. Szabo, Amsterdam – Londres, North-Holland, 1969.

Quine 1972. William V.O. Quine, *Methods of Logic*, 3rd ed., Holt, Rinehart and Winston, 1972.

Vuillemin 1960. Jules Vuillemin, *Mathématiques et métaphysique chez Descartes*, Paris, PUF, 1960.

Vuillemin 1986. Jules Vuillemin, *What are Philosophical Systems ?*, Cambridge, 1986.

Vuillemin 1987. Jules Vuillemin, « Sur la dualité », *Manuscrito*, vol.10, n° 2, 1987.

Éric Audureau, Centre Gilles Gaston Granger
Aix Marseille Université
eric.audureau@wanadoo.fr

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