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Bachelard and Vuillemin, non-Cartesian philosophers? Two criticisms of simple natures.

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Abstract. Both Bachelard and Vuillemin criticised the Cartesian method, with particular emphasis on the central concept of simple nature. However, despite their apparent convergence, these criticisms turn out to be quite distinct: Bachelard gives these natures an ontological dimension and endows them with absolute self-evidence, while Vuillemin proposes an intuitionist and critical interpretation. We show that the first of these analyses is debatable, and does not really reach Cartesianism, and we measure the philosophical implications of the second, which puts Vuillemin on the track of decisionism.

Keywords. Descartes, Vuillemin, Bachelard, French epistemology, intuitionism, algebra.

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§ 1. — Introduction.

Although both Vuillemin and Bachelard were critical of the Cartesian method, what they had in common was that they presented their own philosophical method as a reversal of Cartesianism. The non-Cartesian epistemology that Bachelard introduced in *Le Nouvel Esprit scientifique* (Bachelard 1934) and *La Philosophie du non* (Bachelard 1940), which was to infuse all his philosophy, was in fact the one that, commanded by the new physics, best suited the 'new scientific spirit' on which he insisted so much. Nearly twenty years later, in *La Philosophie de l'algèbre* (Vuillemin 1962), Vuillemin proposed to reform the rules for the direction of the mind introduced by Descartes, in the light of the evolution of modern algebra, from which it was urgent, in his view, to learn lessons in order bring about a 'general criticism of pure reason'. For both, it was clear that the Cartesian method had to be amended, as it had been rendered obsolete by the progress of science.

We know that this method is set out more extensively in the *Règles pour la direction de l'esprit*, in the form of 21 rules, four of which are reiterated in the *Discours de la méthode*, without claiming to be a synthesis. Although Vuillemin refers explicitly to the rules for the direction of the mind in *La Philosophie de l'algèbre*, if only by the wink-wink title of the first part of the conclusion of the work, he focuses in the text itself on the four precepts defended in the *Discourse*, namely that evidence, analyticity, synthesis, and finally the last dealing with the use of the first three. Bachelard, for its part, seems to confine himself to the *Discourse*.

The main target of Vuillemin and Bachelard's criticisms is the first of these precepts: that of self-evidence, which can be linked to the theory of simple natures, on the understanding that only so-called simple natures can be recognised as self-evident only by intuition.[(1)] Descartes gives two non-exhaustive enumerations of these natures in Rules VI and XII of the *Regulae*. Bachelard considers that "the basis of objective thought in Descartes is too narrow to explain physical phenomena", and that this is essentially due to "the initial separation of simple natures" (Bachelard 1934, 142). Non-Cartesian epistemology is thus presented as "a condemnation of the doctrine of simple and absolute natures" (Bachelard 1934, 145). Similarly, Vuillemin points out that "the idea of simple natures as elements susceptible

[(1)] Vuillemin explicitly makes this connection when he reverses Cartesian rules, as is well established in (Maronne 2020, 80) and (Mélès 2020, 21).

of intellectual intuition and of combinations regulated in the genetic method" constitutes "the constant prejudice of his [Descartes] philosophy" (Vuillemin 1960, 182).

§ 2. — The problematic status of simple natures.

The criticism of the thesis of simple natures, which Bachelard and Vuillemin each develop in their own way, involves an interpretation of it that is often implicit. It therefore seems prudent to begin by asking what these simple natures are for Bachelard and Vuillemin, and whether their interpretations coincide. The task is complicated by the fact that Bachelard couldn't react to the publication of *La Philosophie de l'algèbre*, which was published in 1962, the year of his death. Moreover, Vuillemin never refers to Bachelard's criticism developed in *Le Nouvel Esprit scientifique*.

The secondary literature on simple natures agrees on their fundamentally problematic status.[(2)] The long and decisive rule XII, "the conclusion of all that has been said previously", divides these natures into three categories, which can be related to different domains and faculties (Marion 1991): simple material natures, which exist only in bodies, and which come under physics and mathematics, and are grasped by the imagination (figure, extent, movement, etc.); simple intellectual natures, without reference to bodies, which come under metaphysics, and are grasped by the understanding (knowledge, doubt, ignorance, volition, etc.); and finally simple common natures, which exist only in bodies, and which come under physics and mathematics, and are grasped by the imagination (knowledge, doubt, ignorance, volition, etc.).Finally, there are the common simple natures, which can be subdivided into real (existence, unity, duration, etc.) and logical (transitivity of equality, etc.).

Once this tripartition has been established, the concept of simple nature remains no less equivocal. Within the restricted framework of this study, whose aim remains the Bachelardian and Vuilleminian understanding of this concept, we will focus on two of these problems.

The first difficulty concerns the meaning of the term "nature". According to the realist interpretation, these natures designate

[(2)] The study of this aspect of Cartesian doctrine is well developed. Read in particular (Keeling 1937), (Le Blond 1937), (Marion 1975, 1991) and (O'Neil 1972).

ontological constituents of the world, which can be grasped without intermediary by means of intuition. If it's correct, Descartes is assuming a form of direct realism (Keeling 1937). It would then be necessary to conceive of the figure as a real constituent of the world, that of real objects, just as much as the will, which must be understood no longer as a concept, but as an act of will that exists in the world (O'Neil 1972, 168). According to the representative interpretation, on the other hand, simple natures are conceptual in nature. Intuition would then take these simple natures as its object, and they would be distinguished by the evidence they derive from their clarity and distinction. Finally, there are intermediate positions according to which there are two categories of simple natures, one ontological, the other representative (O'Neil 1972). Rule XII seems to justify representative interpretation, since Descartes indicates that his discourse is not about things in themselves, but about the representation of things by the understanding (Marion 1991, 81).

The word 'simple' can also be interpreted in several ways. Natures are said to be simple as opposed to composed, since Descartes states that composition obscures. By simple, we mean elements beyond which analysis cannot go. However, it remains to be seen what prevents us from doing so. Is it, as the former think, because we come up against a simplicity in itself, an absolute, an objectual or conceptual atom, a final element in a series, and therefore a logical impossibility of further analysis? Or is it, as the latter think, a completely subjective simplicity, that which relates to the knowing subject, who push the analysis further without making the notion more obscure and confused? The second interpretation seems to be confirmed by the famous example Descartes himself gives, again in Rule XII: we can certainly analyse the concept of figure in the concept of limit, but doing so would make it more general (it applies to more objects) and therefore more equivocal (these objects are heterogeneous), and therefore less clear and less distinct (Le Blond 1937, Marion 1991). The search for simplicity coincides with the search for clarity and distinction, i.e. evidence, and therefore truth. Analysis is therefore not good in itself, and we must be careful not to push an analysis that would obscurate a notion. All we need to do is analyse and divide until we achieve maximum clarity. So the simplicity in question here seems extrinsic to Descartes.

§ 3. — Bachelard's ontological interpretation.

Towards a non-Cartesian epistemology. Bachelardian's critique of simple natures can be summed up as follows: the Cartesian method seeks to reduce reality to a set simple elements, isolated from each, and is therefore unable to account for the fundamentally relational nature of reality, which has been highlighted by the development of modern science.

In the last chapter of *Le Nouvel Esprit scientifique*, Bachelard attributes to Descartes the belief in "the existence of absolute elements in the objective world", implying that "the elements of reality [are] really given in their integrity" (Bachelard 1934, 146-147). Descartes would thus be led to claim that "the simple object is totally separated from relations with other objects", as a consequence of an "initial separation of simple natures" (Bachelard 1934, 142). On this reading, then, the thesis of simple natures is the main limit of the whole Cartesian epistemology.

Bachelard illustrated his point extensively, in particular by examining the limits of this epistemology in order to grasp the modern concept of mass.[(3)] However, it was no longer Descartes who was targeted, but Newton, who was suspected of also using the Cartesian method and, in so doing, leading to the dead-ends of classical mechanics. Newton's error was twofold: firstly, he reduced the notion of mass to the vague intuition of a quantity of matter, and secondly, he considered that there was a unity to the notion. To maintain that mass is a simple concept that corresponds to a simple nature, we need to consider both that there is only one type of mass and that mass can be an intrinsic property of objects. However, as Bachelard reminds us, relativity goes beyond this simplification and splits the notion of mass by distinguishing between mass in the proper sense, which is indeed an invariant quantity of matter (m_0), and inert (or Maupertuisian) mass, which represents a body's resistance to motion and is itself a function of velocity. It is therefore inaccurate to say that mass is not a constant: it must first be said that it has lost its unit, and that only one type of mass is a constant, while the other remains a function of velocity. Mass is therefore not a simple intrinsic property of objects but a relational property, whose terms cannot be thought of separately: we understand what inert mass is not better but less if we isolate it analytically from velocity.

[(3)] See in particular (Bachelard 1934, 49-52) and (Bachelard 1940, 19-40).

As well as the criterion of analyticity does not imply clarity and distinction, we can no longer claim to go back to the complex, by successive syntheses, from these simple natures. Reality must be grasped "in its value of synthesis" (Bachelard 1934, 147); it cannot be apprehended as an aggregate of simple elements. It is more accurate to say that "mass is energy" rather than that "mass has energy". The first expression establishes a functional link between the two concepts, which can therefore only be understood in relation to each other, whereas the second expression attributes a property to mass. So we have to start from the "primary complexity" of the relationship.

This metaphysical reversal, from beings to relations, is only possible at the price of another reversal, this one epistemological. The simple often dissimulates a simplification, and we must dissociate the basic elements of a theory from the simple elements: a basic element can be quite complex. To try to simplify a complex basic element is to lose clarity and distinction. Non-Cartesian epistemology therefore begins by abandoning the prejudice of simple natures. It assumes a form of desubstantialisation of reality, and transfers the weight of the ontology from beings to relations.

Limits of this criticism. Even if Bachelard's proposal remains of interest from an epistemological point of view, his criticism is rather faulty from a historical point of view. The interpretation of the Cartesian text often seems ambiguous or even questionable.

Bachelard clearly endorse a realist reading of simple natures. In indicating that Descartes' fundamental error is to suppose "the initial separation of simple natures", it is indeed a separation of the ontological constituents of the world that is involved. Yet the Cartesian text seems to see these more as the simplest concepts enabling "the construction an intelligible model of the phenomenal world" (Marion 1975, 135). We shall see with Vuillemin that this interpretation is more convincing.

Moreover, the rule of analyticity is interpreted in the sense of an intrinsic simplicity of simple natures, which leads to hide the role of relations, despite Rule VI. But, as we have seen, it is rather an extrinsic simplicity, from the point of view of knowledge, to which Descartes refers. If isolating speed from mass in order to conceive of them independently obscures them, this is because we are not dealing with simple natures, since what is considered simple, by definition, is that which is optimally clear and distinct. Descartes

does envisage the existence of contingent or necessary relationships between simple natures, which the common simple natures would be responsible for establishing. The question arises, however if there is a necessary link between two simple natures (such as mass and speed), what else is simple about them, since they cannot be conceived separately (LeBlond 1937, 177). It is because he assumes that Cartesian simple nature excludes relations that Bachelard wants to rename it complex nature, but the criticism is only lexical and not epistemological.

In so doing, Bachelard is not far from turning Descartes into a straw man. Even though he does refer to the 'rules for the direction of the mind', he never refers explicitly to the *Regulae*, but always to the *Discours*, about which he readily ironizes (Bachelard 1934, 151).

Towards *Surrationalism*. Descartes' theory of simple natures constrains his method and his philosophy, and it is one of Bachelard's merits that he noticed this. But what can we expect from a non-Cartesianism based on a questionable interpretation of Cartesianism?

For Bachelard, Cartesian ontology is composed of simple natures, understood as the ultimate constituents of reality. Access to these would be possible through the evidence of a clear and distinct intuition. However, Bachelard's critique does not lead him to abandon the idea of founding an ontology, or to dispense with the notion of intuition. Although his critique of Cartesianism questionable, Bachelard nonetheless proposes a relevant way beyond it:[(4)] his non-Cartesianism should not be understood as a reversal of Cartesianism but as a generalisation of it.

The philosopher who wishes to construct an ontology must no longer focus analytically on the search for simple elements, but rather on the complex relationships constituted by the mathematical equations used to describe physical phenomena. This is how a "noumenology" can be constructed, but a non-realist noumenology, one might say. Bachelard points out that these "new phenomena are not simply found but invented, but constructed from scratch". There is therefore an inductive value in equations, which makes it possible to give an ontological status to mathematical idealities.[(5)] Bachelard illustrates this idea with the phenomenon of propagation studied by Dirac: the structure of his equations leads him to

[(4)] This is also the conclusion of (Bontems 2013).

[(5)] I am indebted to the analysis of (Bontems 2010, 57-64).

consider the existence negative-mass entities, an idea that would be difficult to conceive without these equations. To take this ontological suggestion, derived inductively from the equations, seriously is to adopt an attitude that Bachelard describes as *surrationalist*.[6] From the Cartesian intuition of simple natures as direct and obvious access to a reality, we move on to the Bachelardian intuition as attention to the formal properties of an equation that allow us to construct reality.

This is exactly the position that Vuillemin attributes to Descartes and that Bachelard misses. Despite an initial misunderstanding, Bachelard ends up finding it by refining it. Rather than situating Bachelard's approach between realism and idealism, we might propose to interpret it as form of intuitionism, i.e. a philosophy that no longer separates the object of knowledge from the way in which it is known. This is how we can interpret Cartesian philosophy, but also Bachelardian philosophy.[7]

§ 4. — Vuillemin's intuitionist interpretation.

Mathematical intuitionism and philosophical intuitionism. The situation is clearer in Vuillemin. In the many texts he has devoted to Descartes, the latter's philosophy has constantly been interpreted as an incarnation of intuitionism, a term to which Vuillemin has given a precise meaning:

> I give the word intuitionism a meaning close to that which it has received in the philosophy of mathematics. A mathematician is said to be an intuitionist when he requires a proof of existence to provide the means of constructing the object. Similarly, a philosopher is an intuitionist, in the sense used in this book, when he requires the objects of knowledge to show which method legitimizes them. Intuitionist mathematicians argue about the nature and limits of admissible constructions. Similarly, intuitionist philosophers argue about the nature and limits of the methods of knowledge. Kant's deduction of a principle is not Descartes' clear and distinct intuition of an idea (Vuillemin 1994, 7).

[6] See (Bachelard 1940, 33-36) and (Bontems 2010, 61-62).

[7] This is what Vidal-Rosset (1997) proposes.

In this sense, intuitionists use methodological judgements whose characteristic is to make "explicit the mode of cognitive access to the state of affairs" (Vuillemin 1984, 280) and not to propositions that refer to this state of affairs independently of its subjective apprehension. Intuitionists differ according to the type of cognitive access they recognise as legitimate. Descartes would thus have recourse to the subclass of method judgements that Vuillemin calls reflexive, which "manifest the type and intensity of reflection; they express certainty, evidence, doubt, etc." (Vuillemin 1984, 281 n.3). This gives the Cartesian system a pre-critical dimension: a system that explicitly question the limits of human knowledge.[8]

Vuillemin had probably not yet established his classification of philosophical systems, published in 1984 in *Nécessité ou contingence*, when he worked on *Mathématiques et métaphysique chez Descartes* and *La Philosophie de l'algèbre* in the 1960s. However, we feel that this retrospective approach is methodologically legitimate, as the texts devoted to Descartes after the classification remain entirely compatible with the analyses of the 1960s. This reading therefore enriches Vuillemin's interpretation without betraying it.

It is only in this interpretative framework that Vuillemin's interpretation of the simple natures thesis can be understood. It is clearly different from the Bachelardian interpretation in two fundamental aspects. On the one hand, simple natures are conceptual in nature and do have a representative content. The ontological reading contradicts the intuitionism that infuses the Cartesian system, whose ontology consists solely of the actions of knowledge (Vuillemin 1984, 284). On the other hand, these natures are simple in the sense of a maximum of distinction for the knowing subject, and do not claim any simplicity in themselves. A simple nature is recognised by the fact that it possesses an "ideal of perfect distinction between the elements of representation" (Vuillemin 1960, 97). In Descartes' terms, a reflexive judgement of method thus qualifies the act by which the understanding has the intuition of the clear and distinct, that is, of representational contents that analysis reveals to be evident to the subject.

[8] The link between this interpretation and that of Martial Gueroult is well highlighted by (Schwartz 2015, 43-44), in particular Descartes' impossibility of reducing all reality to the clear and distinct. Furthermore, (Maronne 2020, 75-78) highlights the role played by the 4th Cartesian precept, which contains the seeds of the structural method, in Vuillemin's reconstruction of the history of algebraic equations.

Although Vuillemin and Bachelard both criticise the concept of simple nature, they clearly do not conceive it in the same way, and are therefore not fighting the same Cartesianism. If they both assume a non- Cartesianism, it probably does not have the same meaning.

Simple natures in *La Philosophie de l'algèbre*. It was on the occasion of his reflection on the theory of equations that Vuillemin developed the essence of his critique of simple natures in *La Philosophie de l'algèbre*.[9] According to his interpretation, 17th century algebraists treated equations as what Descartes called natures simples: a particularly simple relation between several quantities, which can be intellectually intuited by the understanding (Vuillemin 1962, 232 and 247). Analysis will therefore be the process of "intuition by an attentive mind [which] encounters truths in the form of simple natures" (Vuillemin 1962, 12). The fact that equations are assimilated to simple natures in no way prevents us from conceiving them as fundamentally relational, in accordance with Rule VI. While Bachelard insisted on the incompatibility between the concepts of simple nature and relation, Vuillemin sees this as his archetype in equation theory.

This conception of the equation as simple in nature and of understanding as giving access to the obvious only became problematic over time. As long as it is a question of finding the roots an equation relating its coefficients, for linear, quadratic, cubic or quartic equations, the Cartesian method seems appropriate and the concept of simple nature unproblematic. Admittedly, the solving method becomes more complicated as the degree of the equation increases, but this is a technical problem, not a conceptual one. Vuillemin details the methods Descartes imagined for degrees 3 and 4, in notes VII and VIII respectively of *Mathématiques et métaphysique chez Descartes*.

However, Vuillemin's whole project in *La Philosophie de l'algèbre* is to draw philosophical conclusions about the development of modern algebra, which leads him to question the new difficulty that arise from level 5 onwards. This is because there is no finite algebraic expression for $+$, $-$, $\times$, $/$ and $\sqrt{}$ that produces roots with

[9] Some anticipations of this criticism of Cartesian geometry can be found in (Vuillemin 1960). For a comment, see (Maronne 2020, 73-75).

coefficients for any equation. This expression is of course sometimes possible, as in the example:

$$x^5 - x^4 - x + 1 = 0$$

which gives :

$$(x^2 + 1)(x + 1)(x - 1)^2$$

But then it's a matter of luck, and that's no longer possible for:

$$x^5 - x + 1 = 0.$$

For the fifth degree or higher, it is no longer certain that, starting from the coefficients of the given equation, it is possible to derive its roots. This is the characteristic of a method that Vuillemin calls genetic, which is *a posteriori* and dependent on the contingency of the given. The genetic method, which has been replaced by the structural and axiomatic methods (Mélès 2020), is therefore based on the concept of simple nature. Simple natures, a limit to the progress of algebra and knowledge, can now be eliminated.

As a result, Vuillemin is led to formulate a precept that "rejects the evidence of 'simple natures' that was assumed to be the principle of mathematics" (Vuillemin 1962, 467), and formulates it as follows:

> Only accept a solution as satisfactory if we are able to find it by a truly *a priori* method that eliminates chance in its discovery and shows the reason or cause within a defined structure (Vuillemin 1962, 466).

The work of Abel and Galois,[(10)] in particular, helped us to understand that solving such equations involves objects of a very different nature, namely substitution groups. This method can be considered as more 'pure' in that solutions it allows us to formulate are now entirely independent of the properties of the individuals making up the equation, and result only from the properties of the structures that these individuals instantiate accidentally. The essential, the *a priori* properties of the structures involved, are thus distinguished from the accidental, the *a posteriori* properties of the elements of the equation (Vuillemin 1962, 215).

In this way, both the conception of the equation as a simple nature and that of the understanding as a faculty giving access to

[(10)] Chapters III and IV of *La Philosophie de l'algèbre* are devoted to these topics, respectively.

evidence assimilated to truths collapse. Indeed, analysis no longer leads to comparison of coefficients, but to the study of groups. Now, a group is defined by a series of axioms, and these axioms cannot be considered self-evident by the understanding, and as such cannot be assimilated to simple natures. On the other hand, the mathematician can study the relationship between these axioms, their compatibility, the effects of deleting or adding one of them, etc. (Vuillemin 1962, 468).

Philosophical implications. Vuillemin's critique of simple natures leads him to several of conclusions that clarify the meaning of his non-Cartesianism. Two in particular stand out.

4.0.1. Purifying intuitionism of its dogmatic residues. Vuillemin's main criticism of the Cartesian method is not that it is intuitionist, but that it is genetic. It would seem, then, that in denouncing the latter as a "latent empiricism" (Vuillemin 1962, 118), he is only trying to purify intuitionism of its dogmatic residues. In the context of this discussion, dogmatism corresponds to the doctrine according to which there exists an absolute knowledge independent of any choice that precedes it. It is opposed to pluralism, which holds that all knowledge must be related to the choice that enables it to be established. In other words, dogmatism subordinates practical reason to theoretical reason, whereas pluralism reverses this relationship (Vuillemin 1962, 475-476).

This purification involves a reform of the theory of the faculties. In Vuillemin's reading, Descartes' access to simple natures depends a reflexive judgement of method, the main flaw of which is that it involves to grasp a "passive representative content" (Vuillemin 1962, 470). Galois' theory, however, encourages us to abandon this approach in favour of an active study of the substitution operations that enable resolution. The 4^{th} precept of the new rules for the direction of the mind that Vuillemin introduce invites us to "restore operations instead of simple natures" (Vuillemin 1962, 475). It is why, as "the faculty of thinking a structure" (Vuillemin 1962, 467) that must take the place of understanding when the method is no longer genetic but structural. We thus move from an understanding that intuits a simple relationship between the quantities in an equation in order to derive its roots, to a reason that thinks about the relationships between the axioms that define a structure.

This structural method, which insists on the active nature of operations, is entirely compatible with spirit of intuitionism, whose ontology is made up of the actions of knowledge. This is how, for example, Vuillemin will show how the concept of group made it possible to produce Poincaré's representative space according to purely intuitionist conceptions.[(11)] However, while structuralism may lead naturally to intuitionism, it does not necessarily imply it.

The result is a severe critique of the notion intuition. Its only valid use allow to form the word intuitionism: a metaphysical decision whose choice is produced by reason, which is that of prudence (Vuillemin 1962, 476-479). Vuillemin's non-Cartesianism can thus be seen initially in his attempt not to refute Descartes's intuitionism, but to purify it of its dogmatic residues, the double dogma of simple natures and of an understanding that allows intuition of passive representational contents.

4.0.2. Redefining the concept of philosophical truth. For Descartes, the knowledge acquired by the understanding is absolute, since it is the obvious grasp of the relationship between the coefficients of an equation. From Galois onwards, the solution of an equation is only possible with reference to the addition of arbitrary quantities to a given field, which leads Vuillemin to insist on the "relative character of rational knowledge" (Vuillemin 1962, 247). The solution an equation is no longer given in absolute terms, but is now related to an adjoint field, to an explicitly associated domain of rationality. For example, $x^2 - 2 = 0$ will be reducable on $\mathbb{Q}(\sqrt{2})$, adjunction field of rational numbers specifying $\sqrt{2}$ as an adjunct quantity enabling resolution. Truth can no longer be reduced to evidence: the main prejudice of Cartesian philosophy collapses (Vuillemin 1960, 97).

Vuillemin notes that it's a choice of reason that makes it possible to add this or that quantity to a field, and that this choice is free, a link can be established between decision and truth. However, the aim of *La Philosophie de l'algèbre* is to learn from changes in method in mathematics in order to reform theoretical philosophy. How then can we not be tempted to link this observation about Galois' theory to the decisionism that Vuillemin defended in his own philosophy?

(11) See (Vuillemin 1972, 1994b) and (Thomasette 2020) for a commentary.

Indeed, the philosophical decisionism that can be attributed to Vuillemin[12] finds its expression mainly in (Vuillemin 1984, 1986). However, this thesis can only be understood in the light of two other independent theses:

1. The *structuralist* thesis, which Vuillemin shares with Gueroult, according to which a thesis only makes sense within a philosophical system, i.e. a set of specific concepts and organised assertions,
2. The *classificatory* thesis, which distinguishes Vuillemin from Gueroult, according to which there is a finite number of classes of philosophical systems, within which a potentially infinite number of particular systems can emerge.

The two theses are quite independent: one can be a structuralist like Gueroult, without proposing a classification, or classify philosophies like Kant without subscribing to the structuralism described above.[13] Basically, decisionism brings these two theses together by asserting that it is impossible to prove or disprove the truth specific to a class of philosophical systems, and that without classification, the choice of a philosophy can only be comparative.

The latter is inseparable from a certain domain of rationality, which is the ontology to which the system relates. A philosopher convinced by the relevance of Vuillemin's classification of philosophical systems and pluralism he associates with it is therefore well aware that any resolution of a philosophical problem can only be found within a given system, and not in the absolute, and that this system implies an ontological decision. In the language of Galois, we could say that it is the decision to add a philosophical proposition, which formulates this ontological commitment, to the set of propositions that form the system.

More than a simple analogy, this idea derives from the redefinition of the role of the faculties. It is no longer the understanding that unveils the truth by reducing it to a mere evidence. It is the reason that relates the solution of a problem to a domain of rationality. By studying the method of modern algebra, we can highlight this shift from understanding to reason, from the absolute to the relative. Now that the role of these two faculties has been established, it remains to measure the philosophical consequences.

[12] The term "decisionism", which Vuillemin himself never used, was introduced by (Chauvier 2005).

[13] On the link between structuralism and classification, see (Mélès 2016), in particular p.203-275 concerning Vuillemin's position.

Consequently, just as "[t]he occasion of Platonism was provided by the discovery of irrationals" (Vuillemin 1962, 4), we can consider that the adjunction principle is the mathematical counterpart of Vuillemin's philosophical pluralism. If this hypothesis is valid, it is the purification of intuitionism that would have led Vuillemin to pluralism.

§ 5. — Conclusion.

Despite their apparent convergence, Bache lard and Vuillemin's criticisms of simple natures are very different.

Bachelard seems to miss the target: he gives simple natures an ontological status, fails to see that their entirely subjective simplicity can be identified with an ideal of distinction, and assumes that they are intrinsically non-relational. He sees them as isolated, objective elements of the real world. In attributing to Descartes a form of direct realism, Bachelard seems to have underestimated the pre-critical dimension of the Cartesian project, probably because he did not place the text of the *Regulae* at the centre of his analysis.

Although from a formal point of view Vuillemin also appears to limit himself to examining the four precepts of the *Discours* (Vuillemin 1962, 465 ff.), his critique makes detailed use of the text of the *Regulae*.[(14)] He gives a better account of the spirit of the Cartesian text, which he identifies with an intuitionist philosophy, the core of which is the use of reflexive methodological judgements, characteristic of a critical rather than dogmatic project.

To his ontological interpretation, Bachelard adds a proposal of the same nature: he invites us to stop thinking analytically the reality as a set of simple elements that are joined *a posteriori*, in favour of a form of constructivism whose basic elements are synthetically complex. Following Vuillemin's categories, he becomes the defender of philosophical intuitionism, using constructive judgements of method. On the other hand, Vuillemin's critique of Cartesian method sets him on the path to an early version of pluralism, which leads him away from a particular philosophy. The question is no longer whether we must start from the simple or the complex, but more fundamentally to redefine the role of the faculties and the notion of truth: the understanding no longer reveals evidence, identified with truth, but reason explores the rationality of a structure assimilated to a philosophical system.

[(14)] See for example (Vuillemin 1962, 9, 13, 16, 20, 468, 480).

La Philosophie de l'algèbre therefore ends with the formulation of an authentic non-Cartesian epistemology, whereas the concluding chapter of *Le Nouvel Esprit scientifique* remains basically quite compatible with the spirit of Cartesianism. It is precisely because Vuillemin has brought out more clearly the rationality of the Cartesian system that he has been able to amend it.

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