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Mathematics and cognition

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Mathematics is the language for description of possibilities

A research in biology is the study of living matter. A research in astronomy is the study of celestial matter. A research in chemistry is the study of varieties of matter, their interrelations and interactions. We are observing and measuring something existing in real world, we devise and develop experiments (usually, not in astronomy), and as a result of these actions we construct an explanatory paradigm that for a period of historical time becomes the landmark in the development of scientific knowledge.

But what do we study when we do mathematics?

One possible answer is this: we study ideas that can be treated as if they were realities.

Each such idea ("a mathematical object") should be rigid enough in order to keep its form in every context that it can be used. At the same time, each such idea must be charged with a rich potential of forming connections with other ideas. When an initial complex of ideas is born, connections between ideas might also acquire the status of mathematical objects, thus building a new level of the giant hierarchy of abstractions.

A domain of the lowest level of this hierarchy constitute images of stable material things devoid of all their individual qualities except of their distinguishability from other such things. This leads to the first steps of counting: one, two, three, four... Arguably, another domain of this lowest level is filled by the basic geometric images: a point, a line/curve, a segment,... Contemporary brain studies throw some light at this distinction that correlates with their localisation in neuron nets. But for this essay it is more important, that already at the next floor of our Tower of Babel these domains become bridged: we can start counting some characteristic traits of our mental geometric images, and say that a point has dimension zero, a curve has dimension one, a domain has dimension two. Very soon the resources of our linguistic, "left brain" cognitive images supersede resources of the "right brain", spatio–temporal ones, and what earlier was only a poetical "bridge", now becomes the most serious, mathematical cognitive tool.

Marvellously, it turns out that even abstractions of very high level can somehow mirror reality: knowledge about our world acquired by physicists can be adequately expressed *only* in the language of mathematics.

Nevertheless, a subconscious image of this Tower of Babel often repudiates thinkers with different mind dispositions: believers, philosophers, applied mathematicians, experimental scientists...

In order to better understand how mathematics is applied to the understanding of real world, it is convenient to consider three modalities of such applications: as model, theory or metaphor.

A mathematical model describes (qualitatively or quantitatively) a certain class of possible observations, but prefers not lay claim to anything more than that.

Quantitative models produce very precise predictions, such as predictions of observable movements of planets of solar system.

Qualitative models help understanding such phenomena as stability *vs* instability, attractors (limit states, independent of initial conditions in a certain range), phase transitions that happen when a complex system in its evolution crosses a boundary between two phases (water/ice) or between two basins with different attractors.

Theories differ from models by higher pretensions. The psychological drive of continuous creation of theories is a conception of reality existing independently of material world and raising over it, reality that is knowable only with tools of mathematics.

A mathematical metaphor as a cognitive tool suggests that a certain complex net of observations can be compared with a definite mathematical construction.

A mathematical theory is an invitation to construct working models. A mathematical metaphor is an invitation to think about what we already know. Of course, such a subdivision is neither rigid, nor absolute.

Historically, development of mathematics in a sense is parallel to the development of human languages. Both mathematics and languages serve as bridges between objective reality and its reflection in human collective consciousness.

I am convinced that science, including mathematics ("pure" or "applied") is not a moving force of our civilisation. Thanks to science, we have engines and maps, but science does not decide for us where we should go and where to stop. To believe otherwise means to retreat into times of archaic perception of knowledge as a form of magic, when a person predicting eclipse (or any other development with unclear outcome) was treated as a sorcerer provoking this or that outcome. In fact, the biological function of thinking consists rather in rather *preventing* undesirable outcomes.

Ancient Magi and shamans were also describing the space of potentialities, rather than a direct neighbourhood of a tribe settlement, with its dwellers, movements and needs. Magi did not solve practical everyday problems, it was the function of tribal chief. But the chief lent the ear to his shaman, advisor, who was supposed to find the right thing to do.

The celebrated story that reached us from antiquity is the story of the Croesus, king of Lydia. Preparing a campaign against Cyrus the Great of Persia he decided to ask advice of several most famous Oracles. He accepted the prophecy of Pythia, the priestess over the Oracle of Apollo in Delphi: "If Croesus goes to war, he will destroy a Great Empire". The ambiguity between two possible interpretations of the words "Great Empire" was resolved by his psychology of the war leader. We all know the outcome.

To sum up: mathematics describes the phase space of the real world, the space of its possible developments. It studies the laws, defining various options for development and "initial" or "boundary" conditions: the data necessary of the choice of actual trajectory.