

Does Reason Have Limits?

Review Essay: Stephen Budiansky,
Journey to the Edge of Reason: The Life of Kurt Gödel

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Abstract. Stephen Budiansky has produced the second major biography of Kurt Gödel, after John Dawson's classic study. A comparison is made here between the two books, the latter, a more scholarly exercise — emphasizing Gödel's work — the former intended for a wider audience — emphasizing Gödel's life, in particular his private life. Budiansky, after so much has now been written about Gödel and the intellectual climate of the twentieth century, had an opportunity to reflect on just who Gödel was, on what he represented as one of the towering intellectual figures of not just the twentieth but of any century. Unfortunately, it will be argued, this is an opportunity lost. An attempt will be made, however, to do, in this regard, what Budiansky failed to do — to explain why Gödel to this day remains strangely misunderstood and not fully appreciated for who he really was. His time, it seems, has still not come.

§ — General Remarks.

Stephen Budiansky's new biography of Kurt Gödel complements John Dawson's classic, academically oriented study. As Bernd Buldt writes in a comprehensive review of Dawson, "[He] has resisted the temptation to reach too wide an audience and he doesn't even pretend to be popular ... [His is not] a popular but a scholarly account of Gödel's life and work ..." Exactly the reverse is true of Budiansky. Dawson's is "a very austere ... chronicle of Gödel's life ... [T]here are no statements concerning possible 'influences' on Gödel that made him think or act this way or another. ...

[For example,] Dawson raises ‘an interesting question’ ... regarding ‘[h]ow relevant [his social and educational] milieu’ was to [Gödel’s] own social or intellectual development,’ but refuses to answer it ...”⁽¹⁾ Once again, the exact opposite is true of Budiansky. Again, Dawson frequently interrupts the historical narrative in order to expound, in some detail, on the mathematical background of Gödel’s work, which makes sense, given the purpose of his book. By contrast, given the alternative goal of Budiansky’s, it makes sense that he offers us a continuous, uninterrupted narrative, more likely to grip the general reader than Dawson’s.

Budiansky’s book will, no doubt, enjoy a wide appeal, in part because, unlike Dawson’s, the salient feature of his study is the attention paid to Gödel’s life – in particular, to Gödel’s private life -- as opposed to his work (as evidenced by the titles of the two books). Indeed, Budiansky’s book could have been entitled: *The Private Life of Kurt Gödel*. Whatever one makes of this fact, it remains that it is, as one says, “a good read.” It is well written, far from austere, unified, at times dramatic, historically informed, historically oriented. To describe it as “popular”, however, can be misleading. Budiansky’s narrative is, throughout, serious and thoughtful. The book is clearly written “in a good will” (to borrow from Wittgenstein).⁽²⁾ This, I would stress, is no small virtue.⁽³⁾

Given the importance and prominence of Dawson’s biography, however, and the obvious comparison that would inevitably be made with Budiansky new biography, there is, to my mind, too little attention paid by Budiansky to Dawson’s study – a fact made disconcerting by his odd, seemingly depreciatory reference to Dawson as a “mathematics teacher at a community college”.⁽⁴⁾ Budiansky also makes too little use, I believe, of Hao Wang’s two

⁽¹⁾Buldt 1995, 386. Page numbers refer to a pre-print of this review.

⁽²⁾From the “Forward” to Wittgenstein 1975.

⁽³⁾A virtue sorely lacking in Goldstein 2005, which is popular precisely in the invidious sense of the term. See the devastating review in Feferman 2006, which ends with these words: “Those who are fascinated by Gödel’s theorems ... should not be satisfied with Goldstein’s ‘vast and messy’ goulash: hers is not a recipe for true understanding.” The two biographies to be read about Gödel are by Dawson and Budiansky.

⁽⁴⁾Dawson is Professor Emeritus in the Mathematics Department of the Pennsylvania State College at York, which is how, except for Budiansky, he is always described. I have never seen him referred to elsewhere as a “math teacher at a community college.” Would it be appropriate to refer to Gödel simply as a researcher without students at a small institute in a tiny town in New Jersey?

books⁽⁵⁾ devoted to his conversations with, and thoughts about, Gödel's life and work. (More on this, below.) As for the title of Budiansky's book, I've suggested a possible alternative, but the actual title, *Journey to the Edge of Reason*, invites reflection. Does reason (Plato's and Aristotle's *noûs*) have an "edge"? Can reason be over-used? Can one be *too rational*? Consider Gödel's comments about reason or intellect in a letter to his mother: "I don't think it is unhealthy to apply the intellect [*Verstand*] to any area [whatever] (as you suggest). It would also be unjustified to say that in just this very area [religion] nothing can be accomplished with the intellect. For who would have believed 3,000 years ago that one would [now] be able know how big, how massive, how hot and how far away the most distant stars are ..." ⁽⁶⁾

Is Wang, however, right when, with Gödel in mind, he contrasts the rational with the reasonable: "... [B]eing reasonable seems to include ... a flexibility ... associated with being sensible, and moderate, not excessive. ... [T]he rational should not depart conspicuously from the reasonable. ... [B]eing 'fanatically rational' is, for me, no longer rational."⁽⁷⁾ Can one, however, be "fanatically rational"; can reason be overused? One might think so, remembering Aristotle's emphasis in the *Nicomachean Ethics* on the importance of "moderation", on the so-called "golden mean." If one looks more closely, however, at what Aristotle actually says, it emerges that, for Aristotle, one can't be too *x*, if *x* is a virtue, an excellence. Bravery, for example, is a virtue. One can't be too brave, though one can be too *rash*. (Similarly, *mutatis mutandis*, with vices. One can't be adulterous to the right degree, i.e. "moderately", as Aristotle humorously notes.) What is often described (especially with regard to someone like Gödel) as excessive rationality is thus more aptly characterized as irrationality – as misapplying rationality -- just as what might be characterized as

⁽⁵⁾Wang 1987 and Wang 1996.

⁽⁶⁾Wang 1996, 108.

⁽⁷⁾Wang 1987, 225. Yet Wang himself, it should be noted, says that, "Gödel exemplifies, I think, a way of life and work that inspires greater faith in reason ..." (Wang 1997, IX) Indeed, C. Anthony Anderson has claimed that Alonzo Church's contributions "place him high up among the most important philosophers of this century ... and make a persuasive case for the suggestion that Alonzo Church be taken for the denotation of the definite description, 'The most rational man since St. Thomas Aquinas' – a description which Church is said to have reserved for Kurt Gödel." (Anderson 168–69)

a soldier's being too brave is more aptly described as being reckless or rash. Isn't Gödel right when he comments [in concert with Aristotle⁽⁸⁾] that, "Reason itself doesn't err ... Every error is due to extraneous factors (such as emotion and education)."⁽⁹⁾

Solomon Feferman's characterization of Gödel is, unfortunately, neglectful, or at best unclear, about this distinction. He says that Gödel "had a superb all-embracing rationality which could descend to a maddening attention to detail in matters of everyday life."⁽¹⁰⁾ Grammatically, what "descends" is said here by Feferman to be Gödel's rationality. Moreover, to describe Gödel's rationality as "all-embracing" is itself ambiguous. Is the suggestion that Gödel extended his rationality beyond its proper place? Can rationality itself, however, "descend"? Is there a place where rationality doesn't belong? Wasn't Gödel right when he said to his mother, as we saw, that "I don't think it is unhealthy to apply the intellect [*Verstand*] to any area [whatever] (as you suggest)." More accurate would be to describe Gödel *himself* as sometimes descending from his rationality, as sometimes *misapplying* his intellect, rather than applying it where it doesn't belong. Wasn't Gödel also right when he said, "[i]n our time, *rationalism* is used in an absurdly narrow sense ..."⁽¹¹⁾ In the end, is there really anything surprising, paradoxical, or even noteworthy in the fact that someone can be maximally rational in their work, but at times quite irrational in their life? (Indeed, isn't it a bit of folk lore that logicians and mathematicians are famously known to have more than their share of eccentrics or unbalanced personalities?)⁽¹²⁾

⁽⁸⁾ Aristotle, *De Anima*, Bk I, Ch. 4, 408b 25: "Thus thought and contemplation decay because something else within is destroyed, while thought in itself is unaffected." That Gödel was a close student of such passages in Aristotle is clear from Eva-Maria Engelman 2016, "What is the Link Between Aristotle's Philosophy of Mind, the Iterative Concept of Set, Gödel's Incompleteness Theorems and God?"

⁽⁹⁾ Dawson 1996, 241 (brackets added)

⁽¹⁰⁾ Feferman 1986, 2.

⁽¹¹⁾ Wang 1996, 266.

⁽¹²⁾ In the opposite direction, Feferman (Feferman 98), having stressed Gödel's extreme caution (especially, early in his career) in expressing his true thoughts, draws attention to what seems to be a parallel fact, that "Gödel was very careful in his personal habits, especially as concerns his health", and comments that "the caution discussed here [with regard to Gödel's work] is coherent with other characteristics of his personality." (163). He suggests that "only an in-depth biography could plumb the common sources and establish the interrelationships of the everyday and the scientific personality (for which, see now Dawson 1996)." In fact, Dawson doesn't go very deeply into the possibility of such interrelationships.

§ — Preliminary Concerns.

At the beginning of his Prologue, Budiansky refers to Gödel's "staggeringly brilliant and paradoxical proof that no formal mathematical system will ever capture every mathematical truth within its own bounds." Note first that Budiansky fails to point out that Gödel's incompleteness proofs concern only *consistent* formal systems. An inconsistent formal system, of course, can prove everything expressible in that system, including its own consistency. (A pedantic point, to be sure, but one that's always worth mentioning,) Second, what exactly is *paradoxical* about Gödel's result? Is there something paradoxical in the idea that formalizing an axiomatic system imposes constraints on it? Wang records Gödel being irritated by what he took to be Wittgenstein's interpreting his result as a paradox: "How can Wittgenstein consider it as a paradox if he had understood it?"⁽¹³⁾ Note further that Gödel sharply distinguishes the idea of a formal system from that of an unformalized system of axioms, and emphasizes that the limitations proved by his theorems are a result of the constraints imposed by strict formalization. Wang writes in this regard that "[Gödel's] conception of an axiom system is more liberal than the precise concept of a formal system. ... [H]e would probably say that, although his theorem shows that mathematics is not a *formal* system, mathematics can be captured by an axiom system."⁽¹⁴⁾ Further, Wang comments that "the contrast between formal systems and the axiomatic method as Gödel

Budiansky, by contrast, does. But just this fulfillment of Feferman's psychologistic suggestion or speculation I find suspect. Though clearly Feferman had no intention of demeaning Gödel, such speculations, developed by Budiansky, have the effect of demonstrating that one's hero, after all, had feet of clay, that, for example, Gödel "berated himself for his shortcomings: lying in bed too late in the morning, overestimating himself and underestimating others, 'laughing, talking in a high voice, making faces,' indecisiveness, 'nerves and insecurity in company.'" (Budiansky 2021, 103) Hegel has called out such historical psychologists. "These psychologists," he writes, "are particularly fond of latching on to the peculiarities of great historical figures as private persons. A man must eat and drink, he enters relations with friends and acquaintances, he has feelings and moments of anger [as his valet reveals]. ... Served by such psychological valets in historical writing, the historical personage comes off badly; he is brought down ... even a few degrees below the morality of these fine connoisseurs of humanity." (Hegel 1988, 34–35; brackets added) To be sure, to apply Hegel's comments to Budiansky is, admittedly, a bit too harsh; nevertheless, the general point, I believe, remains.

⁽¹³⁾Wang 1996, 84.

⁽¹⁴⁾Wang 1996, 334; emphasis in the original. It should be added that "Gödel does not take an axiomatic system as an 'implicit definition' of the concepts in it, because it is supposed to be a report of our (generally) incomplete intuitions of

appears to understand them may be seen as a fundamental aspect of what might be called Gödel's dialectic of the formal and the intuitive ... While Gödel is best known for his definitive results on the limitations of formalization, his primary interest is not in what has been formalized but in clarifying our larger intuitions."⁽¹⁵⁾ (More, below, on Gödel's dialectic of the formal and the intuitive.)

On that same page, Budiansky repeats a famous description of Gödel's incompleteness theorem(s) as "the most significant mathematical truth of the century," without explaining who said this, and why. In fact, it was said by W.V. Quine on the occasion of Gödel receiving an honorary doctorate from Harvard University on June 17, 1952. As to what Quine meant by those words, Gödel provided an explanation to his mother. In Wang's words, "[Gödel] points out that the citation should not be taken to say that he is the greatest mathematician of the century, but, rather, that the phrase *most significant* means 'of the greatest *general* interest outside of mathematics."⁽¹⁶⁾

Whereas Budiansky provides too little context concerning Quine's remark, he provides, to my mind, too much historical context concerning Vienna, Brno, and the Austro-Hungarian monarchy, and even the Vienna Circle. Some of what Budiansky has to say about the Vienna Circle in relation to Gödel is valuable,

the concept, and can be revised and expanded." (Wang 1987, 247) This explains why, after the proofs of Gödel and Cohen had established the independence of the Continuum Hypothesis from the axioms of Zermelo Fraenkel set theory, Gödel, unlike most set theorists, believed that the next step was to find (intuitively justified) supplementary axioms that would decide the Continuum Hypothesis. To date, however, no such axioms have been found.

Note that "unformalized", or "informal", does not mean "imprecise". "The ideal of [Gödel's] philosophy," writes Wang, "is to be precise but not technical." (Wang 1987, 239)

⁽¹⁵⁾ Wang 1987, 201.

⁽¹⁶⁾ Wang 1996, 2. Quine, I believe, was right. In Yourgrau 2005, Ch. 4, I argued that the Leitmotif of the 20th century was "formalism", in the most general sense – the dominance of form over content, syntax over semantics, in art, architecture, music, physics, and, crucially, in logic and mathematics. Gödel's theorems (proved formally!) are surely the most important result concerning Hilbert's mathematical formalism, the purest form of formalism of the century. Those who have gone out of their way to point out that Gödel's theorems have not, after all, altered the course of mainstream mathematics may be right, but that's the wrong direction in which to look to assess the ultimate significance of those theorems. Indeed, "[even] among mathematicians who are not logicians," Dawson has commented, "there are still many, including some of the highest stature, who have little awareness or understanding of what Gödel accomplished." (267)

but the amount of detail he provides constitutes, to my mind, overkill. And did we really need, for example, to read on page 21 the minute details of the annual Corpus Christi procession as presented in Joseph Roth's *The Radetzky March*, which include "the blood-red fezzes on the heads of the azure Bosnians burn[ing] in the sun like tiny bonfires", and so on, line after line, for nearly a full page? Indeed, I kept wondering, as I began reading the book, if we would ever get to Gödel.⁽¹⁷⁾ Moreover, this topic has by now surely been over-mined. Besides classics like Carl Schorske's *Fin-de-Siècle Vienna*⁽¹⁸⁾, there are the vivid accounts that have recently appeared by Karl Sigmund⁽¹⁹⁾ and David Edmonds.⁽²⁰⁾ To be sure, Budiansky himself goes out of his way to draw attention to his indebtedness to Sigmund, but that doesn't keep him from repeating much of what Sigmund has already written. Of course, the two books don't completely overlap in this regard, but what overlap there is strikes me as excessive. This is especially disconcerting, since Budiansky notes, ironically, that Gödel himself said, "I don't consider my work a 'facet of the intellectual atmosphere of the early 20th century', but rather the opposite. It is true that my interest in the foundations of mathematics was aroused by the 'Vienna Circle', but the philosophical consequences of my results ... are anything but positivistic ... So my work points toward an entirely different world view ..."⁽²¹⁾ By contrast, arguably, Wittgenstein is, to a much larger extent, a product of Viennese culture, as brought out by Allan Janik and Stephen Toulmin in *Wittgenstein's Vienna*.⁽²²⁾

More disturbing are the detailed excerpts from Gödel's diaries and the notes of his psychiatrist which, though revealing, seem at times too personal, verging on the prurient. Gödel, after all, was an immensely private, reserved, even secretive person. There is something tasteless, now that he's dead and buried, in rifling through papers that he would have been horrified to see exposed to public

⁽¹⁷⁾To be fair, this is a constitutional problem of biographies. Like someone taking as long a run-up as possible to get a good jump off the diving board, biographers typically start their run as far back as possible into the pre-history of their main character, as if learning that his or her great great grandfather was a cloth-merchant who used to spend hours practicing the flute shines reflective light into the distant future.

⁽¹⁸⁾Schorske 1980.

⁽¹⁹⁾Sigmund 2017.

⁽²⁰⁾Edmonds 2020.

⁽²¹⁾Budiansky 2021, 101.

⁽²²⁾Janik and Toulmin 1973.

scrutiny. Indeed, Budiansky himself remarks on “[Gödel’s] dread of revealing his inner self and failings to the scrutiny of others,” but then proceeds, anyway, to reveal Gödel’s inner self and failings to the scrutiny of others, as well as passages from Gödel’s notebooks concerning details of his sexual life with his wife Adele.⁽²³⁾ There is something unclean about this. To be sure, by their very nature biographies tend to be voyeuristic, but Budiansky takes this too far.

By contrast, Budiansky is to be praised for his remarkably balanced portrayal of Adele, who has too often been the subject of mean-spirited caricature.⁽²⁴⁾ Budiansky points out, for example, that Georg Kreisel, Alfred Tarski, and Verena Huber-Dyson “all admired Adele’s high spirits, wisecracking observations about Princeton life, and bantering style with her husband, which he seemed to enjoy.” As Kreisel remarked, “she had little formal education, but a real flair for the *mot juste*.”⁽²⁵⁾ Budiansky might have added something Dawson pointed out, that she was also “a talented seamstress” who “[was] said ... to have made a child’s dress each day of the war for donation to the relief effort. Her contributions were so highly valued that after the war the Viennese gave her a bust of her father (who had died, out of touch with her, during the war years) as a token of appreciation.”⁽²⁶⁾

Budiansky is also to be commended for drawing attention to the fact that “Gödel’s last years in Princeton as a reclusive loner have cast a long shadow over the received portrait of his personality. But that is not the Kurt Gödel his friends knew through his life,”⁽²⁷⁾ nor was that true of the years Gödel spent in Vienna before his move to Princeton. Indeed, nothing could be more unlike the cloistered intellectual atmosphere of the Institute for Advanced Study than the international cultural vibrancy of Gödel’s Vienna, with its fertile coffeehouse culture. Budiansky also brings out the fact that the picture of a somber, unsmiling Gödel (as portrayed on the cover of Budiansky’s book) should not overshadow the fact that Gödel may well have had a dry sense of humor, so dry that it could evaporate before his interlocutor realized what had happened. A good illustration of this is Budiansky’s description of an episode the logician Gerald Sacks describes when he once talked to Gödel about his

⁽²³⁾ Budiansky 2021, 175

⁽²⁴⁾ As in Goldstein 2005.

⁽²⁵⁾ Budiansky 2021, 176.

⁽²⁶⁾ Dawson 1997, 162.

⁽²⁷⁾ Budiansky 2021, 108.

favorite philosopher, Leibniz. Sacks was taken aback when Gödel told him that "... Leibniz was wrong about everything," and then added, "[b]ut it's just as hard to be *wrong* about everything as to be *right* about everything." Was Gödel being serious, or was he pulling Sacks' leg? "He didn't crack a smile," said Sacks, "but I thought, this was a very amusing remark. And I thought, Wow, he cracked a joke. Or did he?"⁽²⁸⁾

Another nice touch by Budiansky is the description he provides of Gödel's lecture style when teaching. We know from Carnap's "Intellectual Autobiography" that Gödel's great predecessor Gottlob Frege's back was usually turned to his students. "He seldom looked at the audience. Ordinarily we saw only his back, while he drew the strange diagrams of his symbolism on the blackboard and explained them."⁽²⁹⁾ Gödel's lecture style was similar, but with one important difference. Budiansky notes that a student at one of Gödel's lectures at Princeton said that "while the lecture was good, Gödel continued facing the blackboard the entire time, even though he did not write on it once."⁽³⁰⁾

As for Gödel's general appearance, Budiansky notes that on arriving by ship in New York in 1933, Gödel was described on the list of arriving alien passengers as "height, five feet seven inches; hair, brown; eyes, brown; complexion, dark."⁽³¹⁾ Curiously, however, in Dawson we're told that Gödel was described on arrival as being 5'7" tall, of light complexion.⁽³²⁾ Dark complexion or light? Who is right?

Concerning the appearance of Budiansky's book, I must say I found the cover quite striking, with a high resolution closeup of a famous, evocative photograph of Gödel.⁽³³⁾ By contrast, Dawson's cover, with an image of a "night-moth", is nothing less than bizarre. True, the name of the nightclub where Adele used to dance was "*Der Nachtfalter*" ("The Night-Moth"), but what has that to do with the subject of Dawson's book, Gödel's *Logical Dilemmas*? (Was Adele's dancing at that club one of the logical dilemmas Gödel had to resolve?) I do have, however, a reservation of another sort with Budiansky's cover. That same photographic portrait of Gödel

⁽²⁸⁾ Budiansky 2021, 105 - 6.

⁽²⁹⁾ Carnap 1963.

⁽³⁰⁾ Budiansky 2021, 159.

⁽³¹⁾ Budiansky 2021, 146.

⁽³²⁾ Dawson 1997, 150.

⁽³³⁾ The resolution of photographs inside the book, however, is disappointing.

adorns the cover of Wang 1987 and is in the background of the cover of Wang 1996. Does not a new biography deserve a new cover? If Budiansky were going to borrow from Wang, it would have been better if he had instead borrowed more of the *content* of Wang's books, which remains, to this day, one of the richest sources of our knowledge of Gödel's philosophy. Moreover, there are striking quotations from Gödel provided by Wang that are cause for reflection. To take just one example, at one point Wang says, "Gödel was in favor of metaphysics and opposed positivism. When I asked him about the work of Saul Kripke⁽³⁴⁾, he said, 'Kripke is, though not a positivist, still doing linguistic philosophy.'"⁽³⁵⁾ This will, I suspect, surprise many people. After all, there was a few years ago an article in the Times Literary Supplement by Arif Ahmed, entitled, "Saul Kripke: The Return to Metaphysics". I think I know, however, what Gödel had in mind with that comment, and to an extent, I agree with him, but that's a discussion for another time.

To be fair, however, Dawson, too, fails to sufficiently avail himself of the rich material about Gödel provided by Wang (though he's far better in this regard than Budiansky). Indeed, there has been a tendency in recent years to be skeptical of the authenticity and/or depth of Wang's reflections on Gödel, and, to my mind, more generally, a lack of appreciation of Wang himself as a philosopher. A good corrective is to be found in a fair and moving portrait of Wang in Mar 2015.

§ — More Serious Issues.

Turning to the technical content of Budiansky's book, one finds several glitches. For example, he states that, "... Russell proposed that the way to think of the number 2 is simply as the definition of all sets that contain a pair of objects."⁽³⁶⁾ As it stands, this sentence makes no sense. Russell didn't propose that the number 2 is a *definition*. He proposed a definition *of* the number 2. And the definition he proposed is that the number 2 is a certain set of sets. Which set of sets? Not simply the set of all sets containing a pair of

⁽³⁴⁾ Kripke, by the way, was for a while a colleague of Wang's at The Rockefeller University, until the university – primarily an institute for scientific research -- closed the department, which contained a who's who of distinguished philosophers, creating a diaspora that enriched a number of other universities.

⁽³⁵⁾ Wang 1996, 138.

⁽³⁶⁾ Budiansky 2021, 108–9.

objects, i.e. the set of 2-membered sets, since that would obviously be circular, defining 2 in terms of 2. Rather, Russell, following Frege, defined the notion of a 2-membered set using purely logical concepts. (After all, his program, like Frege's, is known as logicism.)

More. Concerning the inapplicability of the principle of excluded middle (*tertium non datur*) to infinite sets or sequences, Budiansky writes that, "[Brouwer's] favorite example, one that Wittgenstein later often used, was to ask how one could possibly prove or disprove the proposition that somewhere in the infinite decimal expansion of pi the sequence 7777 occurs. Not finding it could always be answered with the objection that one has just not looked far enough."⁽³⁷⁾ But of course, if the sequence 7777 *does* occur in the decimal expansion of pi, one could prove this simply by calculating the decimal expansion (for which there's an algorithm) and coming upon 7777 after finitely many steps. So, if the sequence 7777 *does* occur, it's *provable* (whether or not in fact proved at a given time.)⁽³⁸⁾ What Wittgenstein actually said was, "In the decimal expansion of pi either the group 7777 occurs or it does not – there is no third possibility.' That is to say: God sees, but we don't know.' But what does that mean?"⁽³⁹⁾ Since Wittgenstein, like Brouwer, is in effect a mathematical verificationist⁽⁴⁰⁾, truth is inseparable from, indeed, is constituted by proof, so for him there is no fact of the matter (even before God's eyes) as to what's "really" going on in the decimal expansion of pi, whether or not we know it. And at a given time, we may have no proof either that 7777 occurs or that it does not occur, but only a proof that it hasn't occurred so far as our calculations have proceeded.

Still more. Concerning the so-called Gödel formula, G, in Gödel's incompleteness proof, Budiansky says, "As Gödel, observed, this means that G is formally *undecidable*: it is impossible to construct with the system a proof of either G or of its negation,

⁽³⁷⁾Op. Cit., 114. It's not essential, but it should be pointed out 7777 was not the series Brouwer mentioned.

⁽³⁸⁾Mark van Atten has pointed out to me that it has in fact been shown that 7777 occurs at position 1589 in the decimal expansion of pi. I thank him, more generally, for helping me to clarify Brouwer's views on this issue.

⁽³⁹⁾Wittgenstein, 1973, 352.

⁽⁴⁰⁾See the illuminating study of Brouwer, van Atten 2004, in which Brouwer's intuitionism is characterized as, in effect, a kind of mathematical verificationism. Van Atten brings out, clearly, how crucial the role of time is in Brouwer's conception of mathematics. Reflecting on this, I've drawn attention in Yourgrau 2019 to what I take to be Brouwer's "time problem."

not-G".⁽⁴¹⁾ He forgot to add the crucial caveat: "assuming the system is consistent". An inconsistent system, of course, *can* prove G, since it can prove everything expressible in the system.

And again. We read, "... [Gödel] was hard at work putting into final form his complete proof of the independence of the Continuum Hypothesis, which he already had published two short notices about in the Proceedings of the National Academy of Sciences."⁽⁴²⁾ Gödel, of course, never did construct a proof of the independence of the Continuum Hypothesis, but rather of its consistency with the axioms of Zermelo Fraenkel set theory.

Still more. Budiansky states, "If the Continuum Hypothesis is both consistent with the axioms of set theory (thus not *disprovable*) and independent of them (thus not *provable*) that would affirm the inadequacy of existing theory ..."⁽⁴³⁾ This should have read: "If the Continuum Hypothesis is both consistent with the axioms of set theory (thus not *disprovable* from them) and also not *provable* from them (thus independent of them), that would affirm the inadequacy of existing theory ..."

Also problematic is that Budiansky gives a strangely brief account of Gödel's writings on Einstein's theory of relativity and the reality of time.⁽⁴⁴⁾ Dawson's discussion, by contrast, is far more complete. And what Budiansky does say is unsatisfactory. He mentions, for example, what Gödel says about the analysis of time in special relativity (STR) without noting that Gödel is contrasting the intuitive, pre-relativistic notion of time with what is called "time" in the formalism of STR⁽⁴⁵⁾. He goes on to say that "there remains in the standard cosmological solutions to Einstein's equations a privileged, and thus in a sense objective, 'cosmic time' ...", without noting that a) the discussion has now moved on from special to general relativity (GTR), and b) that now the contrast is between intuitive time and cosmic time. He continues: "In Gödel's [nonstandard cosmological] model, the universe is not expanding ...", without drawing attention to the fact that Gödel constructed several cosmological models, in some of which the universe is not

⁽⁴¹⁾ Op. Cit. 130.

⁽⁴²⁾ Op. Cit. 210.

⁽⁴³⁾ Op. Cit. 214.

⁽⁴⁴⁾ Op. Cit. 226–27.

⁽⁴⁵⁾ This is an important distinction, since, as I argue below, what Wang described as the dialectic of the formal and the intuitive is an overarching theme of Gödel's investigations.

expanding, and some in which it is. Budiansky then records Gödel arguing that in the non-expanding model, time-travel is possible, which, even if the actual universe is expanding, (which it is) “renders meaningless the notion of the passage of time in any objective sense”, without explaining a) just why time-travel, in itself, rules out the objective passage of time⁽⁴⁶⁾, and b) that at issue here is the intuitive concept of time (which, according to Gödel, implies the passage of time).⁽⁴⁷⁾

Budiansky fails, also, to mention the fact that Gödel employs here a modal argument, from possibility to actuality⁽⁴⁸⁾, an argument form that he employs elsewhere, most famously in his construction of a classic ontological argument for the existence of God, moving from the possibility of God to God’s actuality. As a mathematical Platonist, moreover, Gödel would naturally have endorsed a move from mathematical possibility to mathematical actuality.⁽⁴⁹⁾ Also disturbing is that Budiansky ends his discussion

⁽⁴⁶⁾For an explanation of why, for Gödel, if there is time-travel, there is no time (in the intuitive sense), see Yourgrau 1991 and 2005.

⁽⁴⁷⁾Budiansky’s source for his account of Gödel’s world models, William Bonnor’s 1964 book, *The Mystery of the Expanding Universe: The Humorous, Caustic and Illuminating Story of What Men Think About Their Exploding World*, which contains a very brief account of Gödel’s cosmology, is an odd choice, given the vast literature on this subject that exists today. Indeed, I’ve seen no reference to it in that vast literature. For some reason, Budiansky ignores the classic discussions of Gödel’s cosmology in essays by Howard Stein, David Malament, and Lawrence Sklar, as well as the more recent discussion in Earman 1995, in which Earman responds in part to Yourgrau 1991, even though Dawson pointed out in 1997 that concerning “the ramifications of [Gödel’s] cosmological work for the philosophy of time, [t]o date, only a single volume (Yourgrau 1991) has examined the latter in any detail.” (269) Indeed, even though Dawson remarked again, in 2016: “Surprisingly ... only one extended study of Gödel’s contribution to the philosophy of time (as opposed to the technical papers in relativity theory) has so far appeared: Palle Yourgrau’s *The Disappearance of Time*. (footnote: Yourgrau 1991. Later reissued in expanded form as *Gödel Meets Einstein: Time Travel in the Gödel Universe ...*)” (24–25) That this remains, even today, according to Dawson, the only book devoted to this subject is an indication, I think, of the low repute, or perhaps simply lack of interest, that still attaches to Gödel’s philosophical reflections on time. Especially disconcerting, since Wang reports that for Gödel, in a sense, time is *the* philosophical problem.

⁽⁴⁸⁾Gödel’s modal argument for the ideality of time has proved elusive for many philosophers. Even as rigorous a philosopher of physics as Earman (op cit), for example, has, in my view, mis-formulated it (and thus falsely criticized it). See Yourgrau 1991, 92.

⁽⁴⁹⁾See Yourgrau 1991 and 2005 and van Atten, “Gödel, Mathematics and Possible Worlds,” in van Atten 2015.

by mentioning that Kreisel “thought he perceived ‘between the lines’ of Gödel’s essay a hint of his fascination with the idea of ghosts that could reappear from the past.” I found this reference to Kreisel’s rather flippant comment demeaning of a subtle philosophical argument by Gödel, which is supported by the impressive formal contributions he made to relativistic cosmology. (See below.)

Also problematic is Budiansky’s drawing on the mathematician Abraham Fraenkel’s memoirs⁽⁵⁰⁾ to paint a one-sided portrait of the famous intuitionist, L.E.J. Brouwer -- who had a complex relationship with Gödel⁽⁵¹⁾ -- as “a committed Aryan nationalist” who was “temporarily suspended from his university position as punishment for collaborating with the Nazis during their occupation of Holland” (80). Budiansky is apparently unaware of what Dirk van Dalen has written in this regard in his classic biography of Brouwer.⁽⁵²⁾ Based on van Dalen’s account, van Atten writes that, “[In 1934–1935] Ludwig Bieberbach, a German member of the editorial board of *Compositio Mathematica* who in 1933 had joined the National Socialist movement by entering the SA, asks Brouwer to remove the Jewish members of the board. Brouwer refuses; Bieberbach resigns. ... During the Nazi occupation of the Netherlands, Brouwer assists the resistance by letting them use storage space in the Mathematical Institute, of which he was the director. He offers hiding places to Jews ... in small houses on his premises. In 1943, he advises ... students to sign the declaration of loyalty demanded by the Germans; when some students refuse and go into hiding, Brouwer and Heyting visit them to continue their education.”⁽⁵³⁾ It would have been good if Budiansky had looked into this issue more closely and had drawn attention not only to Fraenkel but also to van Dalen and van Atten.

§ — Avenues Not Explored.

Budiansky mentions, at the end, briefly, that “[Gödel’s] “relativity papers and philosophical insights have begun to receive

⁽⁵⁰⁾ Fraenkel 2016.

⁽⁵¹⁾ For details, see van Atten 2015.

⁽⁵²⁾ Van Dalen 2005.

⁽⁵³⁾ van Atten 2020; brackets added.

considerably more respectful attention than they did from physicists and philosophers during his lifetime.”⁽⁵⁴⁾ This mild statement grossly understates the situation. In point of fact, Gödel’s relativity papers and the temporal idealism he derived from them were, and to a considerable extent still are, shamefully dismissed or derided, or both. Note, to begin with, that Gödel himself, in a bibliography of his writings, listed among his five most important published results, “rotating universes; its mathematical aspect, and its philosophical aspect.”⁽⁵⁵⁾ Einstein agreed. According to Gödel’s friend Oskar Morgenstern, “Einstein told me that Gödel’s papers were the most important ones on relativity theory since his own [Einstein’s] original paper appeared.”⁽⁵⁶⁾ More recently, the distinguished cosmologist G.F.R. Ellis commented that “the issue of the beginning of the universe is still the subject of intense debate; the Hawking-Penrose theorems have set the parameters within which the discussion takes place. Those theorems owe much to Gödel’s papers.”⁽⁵⁷⁾

Yet, as Budiansky himself points out, as late as 1978, at a memorial service for Gödel, Harry Woolf, then Director of the Institute for Advanced Study, in preparing notes for who should speak, wrote, “(minor: relativity – not worth a talk.” Earlier, there had been a more dramatic dismissal. The noted astrophysicists S. Chandrasekhar and J.P. Wright published a paper in 1961 in the prestigious *Proceedings of the National Academy of Sciences*,⁽⁵⁸⁾ in which they claimed they had found an error in Gödel’s results, since he alleged that “time-travel” in one of his rotating universes would take place along a geodesic. As Dawson comments, though Gödel was a member of the National Academy of Sciences, he wasn’t consulted about this challenge, which, it turned out, was mistaken. Yet when the philosopher Howard Stein submitted an article refuting Chandrasekhar and Wright, it was only accepted for

⁽⁵⁴⁾Budiansky 2021, 278 (brackets added).

⁽⁵⁵⁾Wang 1996, 93. Eric Audureau notes that Gödel “considered these ... papers as a whole. ... Thus, from the inner, relative (to Gödel), meaning of ‘Gödel Philosopher’, cosmology should belong to philosophy: his work on general relativity is an application of a philosophy suggested outside of general relativity and obtained on the occasion of thinking about it. ... [H]e believes that the usual cosmological notions of force, inertia, motion, time and so on, are philosophical matters.” (Audureau 2016, 58 - 59)

⁽⁵⁶⁾Moore 1990, 93.

⁽⁵⁷⁾Ellis 1996, 46.

⁽⁵⁸⁾Chandrasekhar and Wright 1961.

publication in 1970⁽⁵⁹⁾, after Gödel himself personally intervened. And the trend continues. As late as 2005, the noted Einstein scholar John Stachel, in a lengthy review of Yourgrau 2005 in *Notices of the American Mathematical Society*, dismissed Gödel's cosmological models as "... an example of that fetishism of mathematics to which some Platonists are so prone."⁽⁶⁰⁾ The distinguished philosopher of mathematics Charles Parsons, in turn, an expert on Gödel, went out of his way to express his approval of Stachel. "Stachel," he wrote, "presents a physically informed discussion whose point of view seems in harmony with what I'm trying to say here."⁽⁶¹⁾

As for the philosophical conclusions vis a vis the ideality of time Gödel believed were supported (NB, not proved) by his cosmological results, these have been accorded a no less unfriendly reception. Stachel, unsurprisingly, had no more patience with them than he had for Gödel's cosmological models. Meanwhile, the distinguished philosopher of science John Earman, after noting the "relative neglect of the philosophical moral Gödel ... himself wished to draw from his famous solution to EFE [Einstein's Field Equations]," concluded that "the neglect has been benign."⁽⁶²⁾ In Yourgrau 2005, I protested that claim by Earman, which prompted Stachel to affirm his solidarity with Earman: "I agree with John Earman's claim – which so horrifies Yourgrau – that the philosophers' neglect of Gödel's views is 'benign'."⁽⁶³⁾ I wasn't the only

⁽⁵⁹⁾Stein 1970.

⁽⁶⁰⁾The moral of Stachel's piece seems to have been, "When it comes to physics, Einstein good; Gödel bad." Yet, as we've seen, Stachel is here contradicting what Einstein himself said about Gödel and relativity. And as for "fetishizing mathematics", the charge might more plausibly have been made against Einstein, in his later years. As Howard Isaacson writes in his recent biography of Einstein, "Now, in [Einstein's] quest for a unified theory, there seemed to be a lot of mathematical formalism, but very few fundamental physical insights guiding him." As Einstein's collaborator Banesh Hoffmann commented, "... the search was not so much a search as a groping in the gloom of a mathematical jungle inadequately lit by physical intuition." While Jeremy Bernstein said it was "like an all but random shuffling of mathematical formulas with no physics in view." (Isaacson 2007, 468). Yet Stachel chose to describe Gödel's physics, as opposed to Einstein's, as "a fetishism of mathematics."

⁽⁶¹⁾Parsons 2010, 182, footnote 32.

⁽⁶²⁾Earman 1995, 194–5; brackets added.

⁽⁶³⁾Stachel, op cit., 865. I should add that I was also "horrified" by Earman's dismissal of Gödel's attempt to grasp the essence of the intuitive concept of time: "This [Gödel's reasoning about the intuitive concept of time] is a pretty piece of ordinary language reasoning. But, like most of its ilk, it leaves one up in the air ... [O]ne can wonder how such intuitions can support such weighty philosophical

one, however, who was put off by Earman's attitude. (Criticism, of course, is one thing; dismissal, another.) The philosopher Mauro Dorato wrote that "[Gödel's] argument is important not only for the conclusion it – unsuccessfully, as we shall see – tries to support, but much more for the brilliant analysis of controversial philosophical notions that it provides; from this point of view, it has certainly *not* received the attention it deserves. Besides Yourgrau's pioneering work on Gödel's philosophy of time [1991, 1999], there are now *two* conflicting reconstructions of Gödel's argument for the ideality of time. Savitt's [1994], which endorses it, and Earman's [1995, pp. 194–200], which rejects it and *somehow considers it unworthy of much attention.*"⁽⁶⁴⁾ And more recently yet, in his review of Yourgrau 2005 in the prestigious scientific journal *Nature*, the logician Martin Davis, an expert on Gödel, dismisses Gödel's argument with no more than a wave of the hand. He writes that "[Yourgrau] argues that because time fails to exist in these universes and because the differences between such universes and our own are accidental, time can't exist in our world either. I doubt many readers would be convinced by this argument."⁽⁶⁵⁾

morals. Gödel's essentialist intuitions are not easy to fathom. ... This game of using an inner sense to perceive conceptual truths is a dangerous one. ... [In his response to Gödel,] Einstein brushes aside the question of the relation of GTR to idealistic philosophy. This seems to me to be the correct response to Gödel." (Earman 1995, 195 - 200; brackets added.)

⁽⁶⁴⁾Dorato 2002, 257; brackets in the original; final emphasis added. Savitt, it should be noted, later reversed his position on Gödel's argument.

⁽⁶⁵⁾Davis, 2005, Vol. 435, 19–20. As far as philosophical refutations go, "I doubt many readers would be convinced by this argument" does not, surely, rank very high. (Note, too, that Davis curiously attributes Gödel's argument to me.) His curt dismissal of Gödel's reasoning based on Gödel's analysis of the essence of the concept of time is surprising, given his important essay, "Why Gödel's Didn't Have Church's Thesis" (Davis 1982). There, Davis sang a different tune with regard to Gödel's search for the essence of an intuitive concept, in that case, the concept of effective calculability. Davis commented that Alonzo Church, in a letter to Stephen Kleene, related that Gödel told him [before the advent of the concept of a Turing machine] that "it might be possible, in terms of effective calculability as an undefined notion, to state a set of axioms which would embody the generally accepted properties of this notion ..." (9; brackets added) Davis went on to remark that "now, this is very much in line with what Turing accomplished. Although his treatment was not axiomatic in any formal sense, he did manage to show that 'generally accepted properties' of effective calculability lead inevitably to a definite class of functions (which turned out to be the same as lambda-definable or recursive functions)." (14)

Is all this, however, just a tempest in a teapot, which Budiansky had no need to take note of in such detail? Am I being prejudiced here, since, so to speak, I have a dog in this fight? I think not. It is indicative of a curious and disturbing phenomenon: the widespread failure to take seriously Gödel as a philosopher. (More on this below.) Saying only, as Budiansky does, that Gödel's philosophical insights have begun to receive considerably more respectful attention than they received previously, does not, I believe, begin to do justice to this disconcerting phenomenon. Why in the world would so many be so willing to dismiss Gödel, of all thinkers, to disrespect (vs. simply to criticize) him when he ventures to go beyond his strictly formal results, or beyond his primary field of research?

To be sure, Budiansky does mention, at the end of his book, the fact that today, "conferences and prestigious publication programs are being devoted to both topics [Gödel's relativity papers and philosophical insights]." ⁽⁶⁶⁾ Once again, however, details matter, and once again, details are lacking. Given the dismissal of Gödel's contributions to philosophy that was a theme of the Boston University Conference in 1995 entitled "Gödel's General Philosophical Significance" that was drawn attention to in Yourgrau 2005 (162–164), Budiansky would have done well to mention a particular recent conference with a very different emphasis, entitled "Gödel's Legacy: Does Future Lie in the Past?", at the University of Vienna, sponsored by the Kurt Gödel Society of Vienna, from July 25 to July 27, 2019, celebrating the 70th anniversary of Gödel's contributions to relativity theory and the 100th anniversary of the confirmation of Einstein's GTR. Budiansky

Indeed, Turing himself made it clear that that he was similarly attempting an analysis of the essential nature of the concept of effective calculability: "In elementary arithmetic," he wrote, "the two-dimensional character of the paper is sometimes used. But ... I think it will be agreed that the two-dimensional character of paper is no essential of computation." (Turing 1936–7, 135). Obviously, Turing was maintaining that the other elements of his analysis *were* essential to the concept. In their attempt to discover the essence of the concept of effective computability, Gödel and Church (and, from his sympathetic account, Davis) were clearly not subscribing to what we saw earlier was Earman's position with regard to Gödel's analysis of intuitive concepts (in Earman's case, the intuitive concept of time): "This is a pretty piece of ordinary language reasoning. But, like most of its ilk, it leaves one up in the air ... [O]ne can wonder how such intuitions can support such weighty philosophical morals" – including such weighty philosophical morals as Church's thesis?

⁽⁶⁶⁾Op. Cit. 278, brackets added.

would also have been well advised to draw attention to the appearance in 2016 of a remarkable collection of essays entitled, *Kurt Gödel: Philosopher Scientist*⁽⁶⁷⁾ (a clear echo of the famous volume edited by P.A. Schilpp entitled *Albert Einstein: Philosopher-Scientist*⁽⁶⁸⁾).

Budiansky would have done well, also, to document more fully Gödel's (in)famous conceptual realism and mathematical Platonism together with his attendant idea of mathematical intuition. The first question here is whether one should believe Gödel when he said that he had been a mathematical Platonist since 1925.⁽⁶⁹⁾ Skeptics about the accuracy of the date of 1925 include the logicians Solomon Feferman, Martin Davis, and Donald Martin. Budiansky refers to this controversy, but fails to mention that Dawson had already drawn attention to it. Dawson pointed out⁽⁷⁰⁾ that the question turns on the correct placement of a comma. The comma occurs in a sentence in a lecture Gödel gave in 1933, which asserted that the axioms "if interpreted as meaningful statements, necessarily presupposed a kind of Platonism, which cannot satisfy any critical mind." Dawson points out that crucial to the interpretation of that sentence is whether the placement of the comma before the word "which" reflected merely Gödel's unfamiliarity with the niceties of English grammar. In his biography, Dawson himself doesn't attempt to resolve the Question of the Comma. Nor does Budiansky.

Buldt, by contrast, in his review of Dawson⁽⁷¹⁾, takes a stab at it. He develops an argument for the thesis that the placement of the comma was simply a grammatical mistake. His first point is based on a kind of principle of charity. This interpretation avoids the conflict that arises otherwise between Gödel's different claims. (And shouldn't one avoid, if possible, charging Gödel, of all people, with inconsistency?) His second point is that "the comma is required by German grammar in the corresponding German sentence" and that in 1933, Gödel, at the time, was probably not fully aware of the fine points of English grammar concerning the proper placement of

⁽⁶⁷⁾Croco and Engelen eds. 2016. To be sure, this important collection is cited in Budiansky's Bibliography, but surely it should have been drawn attention to in the text.

⁽⁶⁸⁾Schilpp, ed. 1949.

⁽⁶⁹⁾Gödel 1933.

⁽⁷⁰⁾Dawson 1997, 100.

⁽⁷¹⁾Buldt 1995.

a comma.⁽⁷²⁾ His third point, Buldt says, is the most important one: “[W]hen in his later years Gödel argued for Platonism the direction of the argument was one from intuition to axioms, justified by these intuitive insights: objects first – axioms second, so to speak. But the platonism he speaks of at the passage in question is of another kind; for in this context Gödel starts out with axioms as uninterpreted and meaningless formulae which are given meaning only afterwards: axioms first – objects second. This way of faking platonism must have been unsatisfying for Gödel. ... [S]uch an added ontology will never enjoy the richness of the ‘original’ one experienced by intuition and will never be able to be more than a mere and much poorer substitute.”⁽⁷³⁾

Buldt’s reasoning, however, is not the last word. The plot thickens when one takes into account a draft of a passage written by Gödel in 1933, cited by van Atten, in which Gödel, having raised some issues concerning Heyting’s intuitionist logical foundations, comments that they are “at least more satisfactory than the ordinary Platonistic interpretation ...”⁽⁷⁴⁾ Notice should also be taken of the defense by Martin Davis (Davis 1998) of his skeptical attitude toward Gödel’s account of the history of his relationship with mathematical Platonism. At the same time, one should consider the reservations concerning Davis’s defense found in Tait 2001, 100–102.

And there’s much more Budiansky could have done to explore the question of Gödel’s Platonism and his concomitant belief in mathematical intuition. For example, he could have cited the curious fact of the widespread disdain, amounting at times to contempt, for this aspect of Gödel’s philosophy, as expressed, for example by Charles Chihara.⁽⁷⁵⁾ He could have pointed, too, to Michael Dummett, who, in his study of Frege’s philosophy of mathematics, gave the back of his hand to Gödel’s attempt to provide an epistemology for his mathematical Platonism.⁽⁷⁶⁾ He refers to Gödel’s “postulation of a faculty of intuition of abstract objects, in analogy with the perception

⁽⁷²⁾Over time, Gödel’s grasp of the subtleties of English grammar increased greatly. When preparing translations of Gödel’s essays to be included in his seminal collection (van Heijenoort 1967), van Heijenoort “... exchanged a total of seventy letters and met twice in Gödel’s office in order to resolve questions concerning subtleties in the meanings and usage of German and English words.” (Dawson, 1997, 217)

⁽⁷³⁾Buldt 1995, 398.

⁽⁷⁴⁾van Atten 2018, 7.

⁽⁷⁵⁾Chihara 1973, Ch. II: “Gödel’s Ontological Platonism”, 60 -83.

⁽⁷⁶⁾Dummett 1991, 301 - 302.

of material objects,” and concludes his discussion by remarking on “the lameness of that analogy.”⁽⁷⁷⁾ “By contrast,” he says, “Frege, in virtue of his logicism, had none of these objections to face.”⁽⁷⁸⁾ He forgets that, Frege, like Gödel, was a conceptual realist, and so he, too, needed to “postulate” such a faculty to account for our access to concepts and other abstract entities, like senses.⁽⁷⁹⁾

More recently, the distinguished set theorist Donald Martin has also dismissed Gödel’s notion of mathematical intuition as a kind of perception.⁽⁸⁰⁾ He cites a famous passage from Gödel in which he says that, “... despite their remoteness from sense experience we do have something like a perception also of the objects of set theory ... I don’t see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception.”⁽⁸¹⁾ Martin objects to the reference, here, by Gödel to the perception the objects of set theory, but he also objects to Gödel’s idea that we have something like a perception of concepts. “[Concerning the] perception of concepts,” he says, at the close of his essay, “... it is hard to see what such perception is supposed to yield that is not yielded by garden-variety understanding of the concept.”⁽⁸²⁾ I’ll comment, shortly, on Martin’s criticism, but I note first that he curiously fails to remark on the fact that the distinguished logician and philosopher Alonzo Church made, in a well-known essay years before Gödel’s remark, almost the exact same observation that Martin finds such fault with. “The preference,” writes Church, “of (say) seeing over understanding as a method of observation seems to me capricious. For just as an opaque body may be seen, so a concept can be understood or grasped. And the parallel between the two is in fact rather close.”⁽⁸³⁾ This remark by Church is especially striking given

⁽⁷⁷⁾Dummett raises a good question about why, if there is such thing as mathematical intuition, proof is so dominant in mathematics. Gödel, however, has a lot to say about this, which Dummett neglects to point out. William Tait, too, in “Truth and Proof: The Platonism of Mathematics” (Tait 1986), has a lot to say about this. One fact, however, should be obvious. Proof ultimately rests on axioms, primitive concepts and rules of inference. These, of course cannot be proved. How, then, is their truth or reliability to be vouchsafed?

⁽⁷⁸⁾Op. Cit. 302.

⁽⁷⁹⁾See Yourgrau 1982 and 2012 and Kripke 2011.

⁽⁸⁰⁾Martin 2005.

⁽⁸¹⁾Gödel 1990, 268.

⁽⁸²⁾Op.Cit. 227; brackets added.

⁽⁸³⁾Church 1951

Wang's comment that, "[Gödel] asserts, in analogy to our perception of physical objects, our ability to *perceive* concepts too. It is necessary to remind ourselves, however, that by this he means primarily our ability to understand and see that certain assertions about concepts are true. He also regards perceiving concepts ... as a kind of *observation*." (84) Indeed, in an essay on Church's contributions to philosophy and logic, C. Anthony Anderson has remarked that "it would be interesting to know if there was any influence, in either direction, between Church and Gödel on these matters." (85)

It is clear, then, that for Gödel, mathematical intuition involves the perception of concepts, and that by perceiving or grasping a concept he means understanding it. In Gödel's words, "[w]e perceive objects and understand concepts. Understanding is a different kind of perception." (86) Yet Dawson, commenting on this view of Gödel's, can still say, after noting Gödel's belief in the possibility of telepathy, that, "[w]hatever one may think of such beliefs [of Gödel's] in extrasensory perception, it is important to note how well they accord with Gödel's belief that mind is distinct from matter, and with his later advocacy of mathematical Platonism". (87) Does Gödel's sympathy with the idea of telepathy really "accord well" with his belief in our ability to understand mathematical concepts? According to Chihara, the answer is yes. "I think," he says, in his critique of Gödel, "we should look for some sort of natural explanation before we turn to postulating nonphysical processes and extra-sensory perception." (88)

(84) Wang 1996, 13; emphasis in the original.

(85) Anderson 1998, 135, footnote 26.

(86) Wang 1996, 235.

(87) Dawson 1997, 30; brackets added.

(88) Chihara 1973, 79. Indeed, "Gödel's reasoning," says Chihara, "could lead to a massive population explosion of our ontology: if we use mathematical intuitions to postulate mathematical objects, it would seem as if we could use 'theological intuitions' to postulate theological objects like angels." (81). He derides the ontological argument for the existence of God, and suggests that Gödel might "express [an] insight by saying that *God is a necessary being* ... [adding] that this axiom forced itself on him as true." (78; emphasis in the original; brackets added). Ironic, since, Gödel, of course, did actually formalize an axiomatic presentation of the ontological argument, which has stimulated a great deal of discussion. Once again, I emphasize how dangerous it is to dismiss out of hand Gödel's philosophical explorations. One can disagree with Gödel's philosophy, of course, but then one can also disagree with Plato, Aristotle, Leibniz, Kant, or Wittgenstein, without thereby simply dismissing them or minimizing their importance as philosophers.

Budiansky, in turn, comments that "... [Gödel] was using the term ["intuition"] to suggest that the human mind can *literally* see mathematical realities through a kind of perception *no different from* the direct sensory perceptions the empiricists decreed to be the only valid basis of physical laws."⁽⁸⁹⁾ Of course, Gödel precisely did not say we can *literally* see mathematical realities. (Was Richard the Lion Heart *literally* a lion? Was he *no different from* a lion?) Gödel says: *a is analogous to b*. Budiansky says Gödel is claiming: *a = b*.

But what about Martin's protest about Gödel speaking of mathematical intuition and the perception of concepts -- as opposed to, in Martin's words, merely "garden variety understanding of concepts?"⁽⁹⁰⁾ We need to keep in mind that Gödel was a conceptual realist, which accounts for his analogy with sense perception. "... [W]e begin with vague perceptions of a concept," Gödel said, "as we see an animal from far away or take two stars for one before using a telescope. For example, we had the precise concept of mechanical procedure in mind, but had not perceived it clearly before we knew of Turing's work."⁽⁹¹⁾ Further, Wang reports that, "according to him [Gödel], it is not the concepts that change, but rather our perceptions of them."⁽⁹²⁾ Of course, if one isn't a conceptual realist, grasping a concept would be understood as mastering the use of a word.⁽⁹³⁾ For Gödel, however, "'trying to see (i.e. understand) a concept more clearly' is the correct way of expressing the phenomenon vaguely described as 'examining what we mean by a word.'"⁽⁹⁴⁾ What Gödel says here should remind one of what another conceptual realist, Frege, had to say. "What is known," says Frege, "as the history of concepts is really a history of either our knowledge of concepts or of the meanings of words. Often it only after an immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in its pure form, in stripping off the irrelevant accretions which veil it from the eyes of the mind."⁽⁹⁵⁾

⁽⁸⁹⁾Op. cit., 266; emphasis added.

⁽⁹⁰⁾I tried to answer this question in Yourgrau 2005, 173 - 4.

⁽⁹¹⁾Wang 1996, 235. Note that sometimes, as Gödel suggests, it turns out there was no one precise concept there in the first place, just as sometimes what we think of as one star is in fact two stars, seen dimly, from far away.

⁽⁹²⁾Wang, 1987, 63.

⁽⁹³⁾Is that what Martin has in mind by "garden variety understanding of concepts"? It certainly was what Wittgenstein and W.V. Quine had in mind. But, of course, Gödel disagrees with such accounts.

⁽⁹⁴⁾Wang 1987, 63.

⁽⁹⁵⁾Frege 1980, vii. Note that as a conceptual realist, Frege, like Gödel, means by knowing the meaning of a word, having knowledge of or grasping a concept.

The two different accounts of what it is to understand the meaning of a word correspond to two different roads leading from Frege⁽⁹⁶⁾: one leads to Gödel and Church and conceptual realism, the other to Wittgenstein, Dummett, and the so-called linguistic turn in philosophy. Gödel, clearly, took the first road. Dummett, like most philosophers indebted to Frege, has taken the second road. (Indeed, I think it's fair to say that Dummett's "Wittgensteinian Frege" has dominated contemporary accounts.) As far as epistemology is concerned, there are arguments in Yourgrau 1982 and 2012 that Frege relied implicitly on something like Russell's notion of acquaintance. Frege, however, never developed that account or accounted for the role of the subject. It's not surprising, therefore, that, in later life, in his search for an epistemology to match his conceptual realism, Gödel turned from Frege to Edmund Husserl, a conceptual and mathematical realist who, in his phenomenology and transcendental idealism, attempted to account for the role of the subject.⁽⁹⁷⁾ Note that by turning toward Husserl's transcendental idealism, Gödel was accepting the idea that for a different kind of object to be grasped, a different kind of (or aspect of) mind was required.⁽⁹⁸⁾

Along the same lines, Karl Popper, interestingly, engaged in what Julius Moravcsik called "Platonistic psychology", arguing from what the mind *knows* to what the mind *is*, as opposed to what could be called "empirical psychology", which argues (based on introspection and behavioral evidence) from what the mind (supposedly) *is* to what the mind is capable of *knowing*⁽⁹⁹⁾.

⁽⁹⁶⁾ See Yourgrau 2005b.

⁽⁹⁷⁾ For an in-depth account and analysis of Gödel's turn to Husserl see van Atten and Kennedy 2003.

⁽⁹⁸⁾ See Tragesser 1977.

⁽⁹⁹⁾ Moravcsik 1981, 109. Similarly, the distinguished Plato scholar Harold Cherniss: "... there are two things in which Plato is more interested than in the theory of ideas itself, for that theory is, after all, only his way of satisfying these two requirements: first, there is such a thing as mind, which can apprehend reality, and second, this reality which is the object of knowledge has absolute and unqualified existence." (Cherniss 1962, 83) (Cited in Wang 1996, 11) And Frege, a conceptual realist, like Gödel, drew attention to the consonance between the (mathematical) mind and its objects: "In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to our reason, and, *as its nearest kin*, utterly transparent to it." (Frege 1980, 115; emphasis added).

Some have argued that this passage demonstrates, rather, the fact that Frege wasn't after all a true Platonist, a conceptual realist, but if Frege wasn't, neither was Plato: "When the soul investigates by itself," Socrates says in the *Phaedo*, "it

Popper⁽¹⁰⁰⁾ echoed Frege in postulating three worlds, the physical, the mental, and a third world of abstract intelligibles, all of which can be “grasped” by the mind. In effect, agreeing with those who are skeptical that the mind, as we know it, could grasp the third world, Popper suggested that we need to revise our conception of the mind. “I suggest,” he says, “that one day we will have to revolutionize psychology by looking at the human mind as an organ for interacting with the objects of the third world.”⁽¹⁰¹⁾

The tendency, thus, to dismiss, ignore, or downplay Gödel’s philosophical explorations is ill advised – and this is true even when Gödel’s thoughts are being expressed in a far from academic context. (So to speak, Gödel is always, as a thinker, “on the clock.”) A particularly interesting example of this can be found in Dawson’s biography. Dawson⁽¹⁰²⁾, mentions Gödel’s letter to his mother in which he says that “had God created in our place beings who did not need to learn, these beings would not just be *we* ... That is, we would not exist at all.” According to Dawson, this reasoning is akin to the controversial anthropic principle, and asserts that “in essence, [Gödel’s] argument begs the question by asserting that God created us as we are because otherwise we wouldn’t *be* what we are, and so *we* wouldn’t exist at all. ... [T]he discussion seems rather naïve for one so philosophically sophisticated.” Once again, however, we have someone being too hasty in dismissing Gödel’s philosophizing. There is no hint in Gödel’s reasoning to suggest that that is *why* God created us. His point is rather that we shouldn’t complain that God could have created a better, indeed, a best possible world, since if God had done so, we – being the kind of beings we are – wouldn’t have existed. Should we be rebuking God for creating us? Far from this being a question begging, naïve argument, it strikingly resembles what Robert Adams, one of the leading contemporary analytical philosophers of religion and a noted philosopher of modality, has written in his well-known essay, “Must God Create the Best?”⁽¹⁰³⁾ (And this in an informal letter by

passes into the realm of what is pure, ever existing, immortal and unchanging, ... *being akin to this*. ... It is in touch then with *things of the same kind*.” (79d; emphasis added) (See, further, Yourgrau 1997, 144, footnote 17).

⁽¹⁰⁰⁾Popper 1962.

⁽¹⁰¹⁾Op. Cit. 127.

⁽¹⁰²⁾Dawson 1997, 211. See also Wang 1996, 106–7.

⁽¹⁰³⁾Adams 1972.

Gödel to his mother!) It is Dawson, it turns out, not Gödel, who is here being philosophically naïve.⁽¹⁰⁴⁾

Skepticism of Gödel as philosopher has even, surprisingly, extended to doubts concerning his appreciation of and understanding of Kant, to the point of characterizing Gödel as “pre-critical”. Yet, though Gödel most admired Leibniz, he took Kant very seriously and studied him closely. As Wang said: “... [Gödel] felt he needed to take Kant’s critique of Leibniz seriously and find a way to meet Kant’s objections to rationalism.”⁽¹⁰⁵⁾ Indeed, “Gödel often compared his work with Kant’s. He knew Kant’s work very well, and spoke highly of his ideas ...”⁽¹⁰⁶⁾ Further, Gödel, as mentioned above, came to admire Husserl, and in Gödel 1995, he argued that in a sense Husserl’s phenomenology represented the fulfillment of Kant’s intentions. Note his closing remark: “But now, if the misunderstood Kant has already led to so much that is interesting in philosophy ... how much more can we expect from Kant understood correctly?”⁽¹⁰⁷⁾ Further, his papers on relativity were inspired by his reflections on Kant: “[my work on rotating universes],” he wrote, “came from my interest in Kant’s views. In what was said about Kant and relativity theory, one only saw the difference, nobody saw the agreement of the two. ... Kant said that the ordinary notion [of time] was wrong and that real time is something quite different.”⁽¹⁰⁸⁾

⁽¹⁰⁴⁾ In fact, there is more than a little wisdom, and not infrequently something quite touching, in Gödel’s letters to his mother. “What you say,” for example, he writes, “about sadness is right. If there were a completely hopeless sadness, there would be nothing beautiful in it. But I believe there can be rationally no such thing. Since we understand neither why this world exists, nor why it is constituted exactly as it is, nor why we are in it ...: why then should we presume to know exactly this is to be all ...? (Wang 1996, 43) And this: “You are completely right that mankind does not become better through the moon flight. ... If the progress in history, legal and political science, philosophy, psychology, literature, art, etc. were as great as that in physics, there would not be the danger of atomic war.” (Wang 1996, 44)

⁽¹⁰⁵⁾ Wang 1996, 164. Indeed, we find Gödel saying, “I have never achieved anything definite on the basis of reading Leibniz. Some theological and philosophical results have just been suggested [by his work]. One example is my ontological proof of the existence of God.” (87; brackets added).

⁽¹⁰⁶⁾ Wang 1996, 171.

⁽¹⁰⁷⁾ Gödel 1995, 387.

⁽¹⁰⁸⁾ Wang 1996, 88. In fact, the longer, unpublished version of his philosophical essay on relativity is entitled, “Some Observations About the Relationship Between Theory of Relativity and Kantian Philosophy”, some drafts of which have been published in Feferman, et al., eds., 1995. For an argument that Gödel’s understanding of Kant, with respect to relativity theory and the ideality of time, was in

§ — The Big Picture.

In addition to not emphasizing, in detail, the curious and disturbing treatment that has often been accorded Gödel's philosophical reflections, Budiansky, more broadly speaking, never attempts to provide a largescale overview of the mainsprings of Gödel's thinking. Too many trees, too little forest. By contrast, to his credit, Dawson, in his biography, does, if only briefly, provide a big picture. Toward the end of his book, he suggests "four deeply held convictions" of Gödel's": 1) "the univ. is rationally organized and comprehensibly to the human mind"; 2) "it [the universe] is causally deterministic"; 3) "there is a conceptual and mental realm apart from the physical world"; 4) "conceptual understanding is to be sought through introspection."⁽¹⁰⁹⁾ One should add one more theme from Gödel that Dawson draws attention to: 5) Gödel's distinctions between internal and external perspectives, which "are crucial to the incompleteness theorems, in which the truth of the undecidable statements can be seen only metamathematically, and to Gödel's consistency proofs in set theory, which rest of the invariance of certain notions under internal relativization. Long before anyone else, Gödel saw and clearly understood the need for making such distinctions."⁽¹¹⁰⁾

Dawson's list, I think, is good, but I would supplement it by adding the following: 6) what I've called (by contrast with the Hilbert program) the Gödel program, the attempt to determine the limits of formal methods in capturing intuitive concepts,⁽¹¹¹⁾ something we see evidenced in his completeness theorem, his incompleteness theorems, and his writings on Einstein and relativity. Notably, in all but the first case, the result was negative. Another positive example, however, and, according to Gödel, a very surprising one, of the capturing of an intuitive concept by formal means was Alan Turing's analysis of mechanical computability.

fact subtle and sophisticated (contrary to what is suggested in Parsons 2010), see Yourgrau 1999, 115–122.

⁽¹⁰⁹⁾Dawson 1997, 261.

⁽¹¹⁰⁾Op. cit., 263–4.

⁽¹¹¹⁾Yourgrau 2005a, 75. Van Atten has used the idea of the Gödel program to help locate Gödel with respect to Brouwer and Hilbert: "A particularly attractive topic is what has been called by Palle Yourgrau (2005, 75) 'the Gödel program'. While Hilbert focused on the formal aspects of mathematics, and Brouwer on intuitive aspects, Gödel focused on the interplay between these two aspects ..." (van Atten 2015, 83.) (Recall, in this regard, Wang's comment about Gödel's dialectic of the intuitive and the formal.)

Finally, I would add 7) the dominant role of conceptual vs. combinatorial thinking in Gödel's investigations. Note, for example, a remark Buldt makes in his review of Dawson: "These statements are not intended to dwarf Gödel's mathematical brilliance. Rather, they are intended to highlight that his genius was in conceptual issues."⁽¹¹²⁾ Budiansky, too, I was glad to see, draws attention to this crucial aspect of Gödel's thinking. He provides a revealing quotation from a notebook of Gödel's: "I am apparently untalented nor interested in combinatorial thinking ... I am apparently talented and interested in conceptual thinking ... Therefore, I should dedicate myself to the foundations of the sciences (and philosophy)."⁽¹¹³⁾ Budiansky, however, fails to expand on this theme, except to make it an occasion for a psychological speculation: "... in a world that would often fill Gödel with anxieties, the realm of pure mathematical logic beckoned with the enfolding reassurance of certainty, so noticeably absent everywhere else, except perhaps in the atavistic feelings of childhood security that he would at times desperately cling to ..."⁽¹¹⁴⁾

This was an opportunity lost for Budiansky to have provided a wide-angle view of Gödel as a thinker. He could, for example, have placed alongside that quotation from Gödel's notebook what the distinguished mathematician Paul Cohen -- whose proof, combined with Gödel's, established the independence of the Continuum Hypothesis -- said in his essay, "My Interaction with Kurt Gödel: The Man and His Work".⁽¹¹⁵⁾ Pondering Gödel's incompleteness theorem, he remarked that he once thought to himself that "... the proof was basically *simple*, when stripped to its essentials" (xiv)⁽¹¹⁶⁾, though he added that "I felt that its discoverer was above me and other mere mortals in his ability to

⁽¹¹²⁾ Buldt 1995, 373.

⁽¹¹³⁾ Dawson 1997, 76.

⁽¹¹⁴⁾ Ibid. Some will no doubt find such psychological speculations convincing. I'm not one of them. If memory serves, for example, during some of the saddest periods in Mozart's life, he wrote some of his "happiest" music, while in some of his happiest periods, he wrote some of his "saddest". In any case, in the matter of psychological speculations concerning Gödel and mathematical Platonism, Budiansky is not alone. Dawson suggests that Gödel's Platonism may be a sign of "arrested development." (Dawson 1997, 2) Surely, this is going too far.

⁽¹¹⁵⁾ Which introduces his monograph, Cohen 2008.

⁽¹¹⁶⁾ Gödel, by the way, agrees: "My work is technically not hard. One can see why my proofs work." (Wang 1996, 85) Interestingly, Feferman seems to disagree, at least in part, writing that "[c]oncerning Gödel's methods, one may say that many of the constructions and arguments were technically difficult for their time, or at any rate, too novel or unexpected to be readily absorbed ..." (Feferman 1986, 33)

understand what mathematics – and even human thought for that matter – really was.” (xiv) He went on to comment that “... logic seemed to me to have little of the element of *combinatorial* thinking – or more crudely put, *cleverness* – that I found in analysis and number theory.” (xiv). The contrast, here, between the philosophical logician and the pure mathematician with regard to *conceptual*, or *philosophical*, thinking vs. *combinatorial* thinking is something worth pondering. Including Cohen’s remark that, in his youth, “I felt it [Gödel’s Incompleteness Theorem] was probably primarily a theorem of *philosophical* interest, having little or nothing to do with the concrete questions of number theory that so infatuated me in those years.” (xiii) (Later, however, he had wondered “[h]ow can someone thinking about logic in almost *philosophical* terms discover a result that had implications for Diophantine equations?” (xiv) (all emphasis above, added)

Indeed, it’s striking how often Cohen mentions in that essay the crucial role played by philosophical thinking in Gödel’s logical/mathematical work, in contrast to Cohen’s own attitude to philosophy: “... by temperament, even to the present day, I could not really be involved in *philosophical* controversy.” (xiv). And there’s still more that’s relevant from Cohen. “His [Gödel’s] approach in discovering the notion of a constructible set seems to have been decidedly influenced by the *philosophical* discussions of the past ... I found the tone of his article difficult to understand, as it seemed to hover between *philosophy* and mathematics.” (x) “His [Gödel’s] ideas [about the Continuum Hypothesis] were grounded in a deep *philosophical* belief as to what the human mind could achieve.” (xiii) “[T]here seemed no way for me to find an entry into the kind of deep *philosophical* thinking that I so admired in Gödel and translate that into concrete mathematics.” (xv) “I read Gödel’s article on the continuum problem (Gödel, 1947) ... [T]o a large extent it was not even comprehensible to me. For one thing, it assumed a certain *philosophical* point of view and the *philosophical* ramifications of various attitudes toward it. I had never been attracted to *philosophy* ...” (xv) (all emphases added)

Surely, the strikingly contrasting relationship between Gödel and Cohen in this regard is worth further exploring,⁽¹¹⁷⁾ as is, by

⁽¹¹⁷⁾Recall the discussion above on Gödel’s response to the question of whether to accept Church’s Thesis concerning effective computability. As Feferman put it, “Gödel was unconvinced by Church’s thesis, since the proposed identification of the effectively computable with the lambda-definable functions did not rest on

contrast, Gödel's kinship, here, with Frege, whose logical thought also centered around the consideration of (what today we would call) concepts.⁽¹¹⁸⁾ Frege's seminal monograph that gave birth to modern formal logic, after all, was entitled, *Begriffsschrift*, or *Concept Writing* [or *Conceptual Notation*], and the subtitle of his monograph, *The Foundations of Arithmetic* (1980) (the *Grundlagen*), was: *A Logico-Mathematical Enquiry into the Concept of Number*. Notice how many

a *direct conceptual analysis* of the notion of finite algorithmic procedure. [Gödel] always referred to Turing's work as having provided the 'precise and unquestionably adequate definition of formal system' by his 'analysis of the *concept* of 'mechanical procedure' ..." ("Kurt Gödel: Conviction and Caution", Feferman 1998, 162; emphasis added) Again, one sees why Gödel became interested in Husserl's idea of categorial intuition.

In this regard, Feferman poses a good question: "... one may ask why Gödel did not pursue such an analysis *himself*. ... My guess is that he also feared that no such proposal could be made convincing to the mathematical public of his day, just as the concept of truth would not be taken seriously. If so, the subsequent development showed Gödel to have been mistaken. Though certainly there were controversies about both Tarski's analysis of truth and Turing's thesis, they eventually took their place as accepted cornerstones of mathematical logic." (162–3).

Yet precisely one of the controversies still surrounding Tarski's analysis of truth is its relevance to the *intuitive* concept of truth. As for what a reasonable assumption would have been about the reception during the early years of Gödel's work of an account of the concept of truth, see Dawson 1997 (59): "... in discussing Tarski's epochal paper on the concept of truth in formalized languages, Carnap commented that when he invited Tarski to speak on the subject at the September 1935 International Congress for Scientific Philosophy, 'Tarski was very skeptical. He thought that most philosophers ... would be not only indifferent, but hostile to the explication of the concept of truth.' And in fact, 'at the Congress it became clear from the reactions to the papers delivered by Tarski and myself that Tarski's skeptical predictions had been right ... There was vehement opposition even on the side of our philosophical friends.'"

That was then, however. How about now? I have been trying to demonstrate in this Review that with respect to Gödel's entire career, early and late, one should be skeptical of Feferman's claim that "... Gödel's spreading fame should have reassured him that he would not be laughed offstage if he were to go beyond the purely (logico-)mathematical formulation of his results." (Feferman 1998, 163) Similarly, we should think twice about Hao Wang's "mention[ing] a number of times some of the features of Gödel's temperament that led to [his] isolation ... [including] his tendency to overestimate the hostility that his own views would encounter." (Parsons 1998, 19–20; brackets added.)

⁽¹¹⁸⁾Frege refers to functions, which he calls "unsaturated", to contrast them with objects (like numbers and sets). A function whose value is always a truth-value he calls a concept. As pointed out in Feferman 1998, concepts in this sense, are, for Frege extensional, in contrast with what Frege calls "senses", which are intensional. (For an investigation of the complexities of Gödel's conception of concepts, see Crocco 2006 and 2016, and Yourgrau 1997, 162–63, footnote 29.)

times Frege, too, invokes concepts in the *Grundlagen*. The conclusion, so called, of his crucial “Relativity Argument”⁽¹¹⁹⁾, which addresses the question: what do numbers (i.e. the natural numbers, the counting numbers) number?, is that numbers are assigned only to *concepts*, which are the units of counting. As for defining the numbers themselves, he employs the crucial notion of equinumerosity, which relates *concepts* in terms a one-one correspondence between the members of (in effect) their extensions. Finally, according to Frege, the number assigned to the *concept* *c* is itself defined as: the extension of the *concept*, equinumerous with the *concept* *c*, i.e. as a set of *concepts*.⁽¹²⁰⁾

Returning to Gödel, keep in mind that his explorations into relativistic cosmology were at the service of his attempt to assess the limitations of the “formalism”⁽¹²¹⁾ of relativity theory to capture the intuitive concept of time. Similarly, his approach to securing the foundations of mathematics in contrast to the Hilbert program’s emphasis on syntax and consistency, focused on the clarification of meaning of *concepts*. Thus Dawson⁽¹²²⁾, quoting Gödel 1995: “the certainty of mathematics is to be secured not by ... manipulation of physical symbols, but rather by cultivating ... knowledge of the abstract concepts themselves.” Which points, once again, to Gödel’s interest in Husserl and categorial intuition⁽¹²³⁾ and to

⁽¹¹⁹⁾For details see Yourgrau 1997.

⁽¹²⁰⁾Ironic, since Frege insists that numbers are objects, and that the fundamental distinction in ontology is between objects and concepts (more generally, functions), yet defined a number as a set of *concepts*, i.e. as an object whose only members are non-objects! By contrast, Russell basically followed Frege’s definition, but defined a number as a set of *sets*. (Gödel, it should be noted, agreed with Frege’s ontology. “Every being is for Gödel,” said Wang, “either a concept or an object ...” [Wang 1987, 309])

⁽¹²¹⁾“Formalism” in a general, mathematical sense, not the strict sense involved in the notion of a formal system of logic.

⁽¹²²⁾1997, 219.

⁽¹²³⁾Interestingly, Gödel’s optimism about Husserl’s methodology was countered by Wang’s skepticism. “Gödel seems to suggest (as an ideal),” wrote Wang, “that we should aim to use the phenomenological method to discover the axioms for the primitive concepts of philosophy ... But I am not aware of any conspicuous example of definite axioms arrived at in this manner. Neither the axiom of choice, the axiom of replacement, the ‘axiom’ of constructability, the ‘axiom’ of determinacy, nor even Dedekind’s axioms for arithmetic were obtained by going back to the ultimate acts and contents of our consciousness in the manner recommended by phenomenology.” (Wang 1996, 159) No doubt Frege would have been sympathetic with Wang’s skepticism. “Do the concepts,” he wrote, “as we approach their supposed sources, reveal themselves in peculiar purity? Not at all. It is as

the radical difference between the Hilbert program and the Gödel program, as well as to the striking contrast between Gödel's understanding of mathematics and Wittgenstein's. "In mathematics," said Wittgenstein, "*everything* is algorithm and *nothing* is meaning."⁽¹²⁴⁾ Though physically their paths crossed only once⁽¹²⁵⁾, philosophically, Gödel and Wittgenstein collided. Gödel himself spent scant time on Wittgenstein, but Wittgenstein devoted a great deal of attention to what he saw as the necessary demythologizing of Gödel's incompleteness theorems. I agree with Juliet Floyd when she says that "... one of the most fascinating and extreme philosophical clashes of the century is that between Wittgenstein and Gödel."⁽¹²⁶⁾ Given that Wittgenstein is widely considered to be one of the greatest, if not the greatest, philosophers of the twentieth century, Floyd's remark suggests that Gödel, too, should be taken seriously as a philosopher, whereas, as I've been arguing throughout this Review, quite the opposite has in fact been the case.⁽¹²⁷⁾

though everyone who wished to know about America were to try to put himself back in the position of Columbus, at the time when he caught the first dubious glimpse of his supposed India." (Frege 1980, vii – viii) At issue, here, it should be emphasized, is not whether ultimately our confidence in the fundamental concepts and axioms is based on some sort of mathematical intuition (though Frege would no doubt resist using the term, "intuition", here), but rather on what is the best method to help us arrive at these foundational intuitions -- indeed, on whether there even is a "philosophical method" to be found here.

⁽¹²⁴⁾ Wittgenstein 2005, 468, quoted in Juliet Floyd 2002.

⁽¹²⁵⁾ The occasion was a famous lecture that Brouwer gave in Vienna. "I was never introduced to Wittgenstein, and have never spoken a word to him," said Gödel. "I only saw him once in my life when he attended a lecture in Vienna. I think it was Brouwer's." (Wang 1987, 49)

⁽¹²⁶⁾ Floyd 2002, 287.

⁽¹²⁷⁾ Besides the explicit criticisms of Gödel as philosopher I've drawn attention to, there is also Gödel's failure to engage in debate with most of his philosophical contemporaries. Thus Parsons comments that "I argue elsewhere that Gödel, in his reliance on his notion of concept, does not really have an answer to the deeper Quinean criticism of the ideas about meaning that underlie the analytic-synthetic distinction." (Parsons 1998, 15) Indeed, according to Parsons, there is "a serious weakness in Gödel's engagement with philosophy in the latter part of his career: his largely self-imposed isolation after 1940 from the philosophical community around him ... [B]oth [Gödel and Wang] felt themselves outsiders in the contemporary philosophical world." (19; brackets added) To an extent this is all true, but Parsons neglects to mention Gödel's extensive engagement with the "philosophical community" that did not surround him, including, in particular Husserl. Indeed, until recently, there has been a widespread failure of the philosophical community that surrounded Gödel, including Quine, to engage with the philosophical community that included Husserl. One could add, further, that Gödel's colleague, Church, a fellow conceptual realist, did engage, in detail, famously,

I'm puzzled, therefore, by Floyd's remark that "... we ought not to defer to Gödel's authority as a philosopher simply in virtue of his greatness as a logician."⁽¹²⁸⁾ Who, exactly, is supposed to be deferring to Gödel's greatness as a philosopher simply in virtue of his greatness as a logician?

§ — Who Was Kurt Gödel?

In any case, it's clear that Gödel was a logician with a decidedly conceptual/philosophical orientation. Considered, however, purely formally, as a mathematical logician, how should Gödel be ranked? John von Neumann described him as "the greatest logician since Aristotle." True, Aristotle invented logic, but a better comparison would be to Frege, who developed logic into the mathematical science we know now. Not for nothing is van Heijenoort's seminal collection of original papers on mathematical logic entitled *From Frege to Gödel*.⁽¹²⁹⁾ Wang, however, puts it slightly differently: "Nobody can deny that [Gödel] is the greatest logician of the century."⁽¹³⁰⁾ Best, however, I believe, would be to describe Gödel, in this regard, simply as "the greatest logician of all time", as I did in Yourgrau 2005a. Stachel, however, considers that description over the top, complaining that "[in Yourgrau 2005a] both Gödel and Einstein are described in over-the-top superlatives".⁽¹³¹⁾ Yet Saul Kripke, whose decision on this question is surely decisive, has described Gödel as "the greatest logician of all time".⁽¹³²⁾ That is strong praise, indeed. But does it capture who Gödel really was?⁽¹³³⁾

with Quine. (See Anderson 1998.) Perhaps Gödel believed that he had no need to add to what Church had accomplished in this regard.

⁽¹²⁸⁾Op. cit., 288.

⁽¹²⁹⁾Van Heijenoort 1967. Similarly, best would be to compare Einstein to Newton. Gödel himself said that "the beginning of physics [as we know it] was Newton's work of 1687." (Wang 1996, 167; brackets add)

⁽¹³⁰⁾Wang 1987, 168.

⁽¹³¹⁾Stachel 2005, 862, brackets added.

⁽¹³²⁾Kripke 2013, 95, footnote 3.

⁽¹³³⁾Compare: who really was the Canadian pianist, Glenn Gould? The violinist Yehudi Menuhin, himself a major artist, described Gould as "this great phenomenon ... [who] presented a great rift in the continuity of the expected and the ordinary." (McGreevy 1983, 303) In other words, Glenn Gould was not just a gifted pianist. He was a "historical discontinuity". Is not the same true of Kurt Gödel, who was not simply a gifted logician?

Was Gödel, for example, also one of the most significant *philosophers* of the 20th century?⁽¹³⁴⁾ Interestingly, in Wang 1987, one finds the following comment: “It is common to view Gödel as a mathematician. ... It is, however, more appropriate to view Gödel as a ‘philosopher-scientist’ [echoing the title of the famous Schilpp volume on Einstein⁽¹³⁵⁾].”⁽¹³⁶⁾ In the years immediately following his book, however, few took up Wang’s suggestion. Indeed, in Yourgrau 2005, I commented on a 1995 Boston University conference devoted to Gödel’s contributions to philosophy, where prominent speakers were clearly irritated at Gödel’s philosophical pretensions, which, the claim was, were taken seriously only because he was a great logician.⁽¹³⁷⁾ Apparently, my critique of that attitude succeeded in causing further irritation. In that book, as in Yourgrau 1999, I had been making a case for the idea suggested by Wang of taking Gödel to be, in essence, a philosopher-scientist, focusing on Gödel’s philosophy of time -- just one aspect of Gödel’s wide philosophical interests, though an important one⁽¹³⁸⁾, and illustrative about what made Gödel’s philosophy special. The winds, however, have finally, begun to shift, if only a little. In 2016 Wang’s suggestion finally came to a kind of fruition with the publication of a volume entitled, *Kurt Gödel: Philosopher Scientist*⁽¹³⁹⁾, centered around Gödel’s newly transcribed and translated Max. Phil.⁽¹⁴⁰⁾ Notebooks. Surely, Budiansky should have drawn attention, in some detail, to the changing climate with regard to Gödel’s

⁽¹³⁴⁾ Buldt, it should be noted, admires Dawson’s austerity in his biography and praises him for precisely *not* doing what I’m recommending here. In Dawson, says Buldt, “there are no sweeping claims about Gödel’s greatness or exaggerations about the significance of his results.” (Buldt 1995, 387) “Dawson,” he continues, “avoids a growing ‘Gödelmania’ ... in the public.” I, too, appreciate Dawson’s austerity, his extreme carefulness, his fact-based approach. That is one of the virtues of his book. At the same time, at some point, I believe, it behooves one to try to determine who Gödel really was, what his place is in cultural history. And is not a biography the right place in which to attempt to determine this? Is not one of the main reasons why one reads a biography to find out who someone really is?

⁽¹³⁵⁾ Schilpp, ed. 1949.

⁽¹³⁶⁾ Wang 1987, 1; brackets added.

⁽¹³⁷⁾ I was, frankly, taken aback by this attitude toward Gödel. I wasn’t alone. At the conference, I was seated next to the distinguished astrophysicist from the Institute for Advanced Study, Piet Hut, who whispered that he couldn’t believe what he was hearing.

⁽¹³⁸⁾ Indeed, as mentioned earlier, according to Wang, Gödel believe time to be in a sense *the* philosophical question.

⁽¹³⁹⁾ Crocco and Engelen eds. 2016.

⁽¹⁴⁰⁾ I.e. Philosophical Maxims.

reputation as a philosopher-scientist. To be sure, as mentioned, at the end of his book, Budiansky does comment that Gödel's formal and philosophical writings on relativity are beginning to receive "respectful attention", but a) he gives no details about this, and b) he doesn't mention the respectful attention Gödel is beginning to receive for his more extensive philosophical reflections.

As for Gödel's incompleteness theorems, which, like Einstein's theory of relativity, enjoy a life of their own in cultural folklore, Budiansky notes, correctly, that "probably more wrong things have been said about [Gödel's] proof than any other mathematical theorem in history, a dubious distinction – if perhaps an inevitable consequence of popular fame in a field such as mathematics."⁽¹⁴¹⁾ But, once again, he fails to provide details concerning where to look to find a good corrective to the abuse those theorems have suffered. I would strongly recommend, in this regard, the excellent study, written approachably but on a high level, by Torkel Franzén, *Gödel's Theorem: An Incomplete Guide to Its Use and Abuse*.⁽¹⁴²⁾ Surely, a serious biography such as Budiansky's should serve, among other things, as a guide to how to resist the abuse to which Gödel's theorems have been subjected, as well as how to find an illuminating and rigorous presentation of the theorems themselves.

In conclusion, I've said a lot here about what Budiansky might have done, or should have done, in his new biography of Gödel, especially in light of what Dawson has already accomplished in his now classic study, but it remains that what Budiansky has actually done is significant and commendable. He has produced an engaging historical narrative that will no doubt grip the imagination of the general reader and, at the same time, provide new insights into the private and professional life of Kurt Gödel from which both experts and neophytes will profit. And to repeat what I said at the beginning: the book is clearly written "in a good will", and this is no small virtue.⁽¹⁴³⁾

⁽¹⁴¹⁾Budiansky 2021, 278. Indeed, notes Budiansky, "like Heisenberg's Uncertainty Principle and Einstein's Theory of Relativity, Gödel's Incompleteness Theorem has provided what Alan Sokal and Jean Bricmont ... called 'an inexhaustible source of intellectual abuse' ..." (278) In fact, those three epoch-making discoveries have something more fundamental in common. "Einstein, Gödel, Heisenberg [were] three men whose fundamental scientific results opened up new horizons, paradoxically, by setting limits to thought or reality. ... Mysteriously, each established an ontological conclusion about *reality* through the employment of an epistemic principle concerning *knowledge*." (Yourgrau 2005a, 3; brackets added)

⁽¹⁴²⁾Franzén 2005.

⁽¹⁴³⁾I thank Mark van Atten for his helpful comments on this Review.

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