Annals of Mathematics and Philosophy
Online version, 8/04/2024

$M \times \Phi$

Lautman, mathematical critic

Fernando Zalamea

Abstract. We offer a new perspective on Lautman's work, envisioning him as a "mathematical critic" rather than a so-called "philosopher of mathematics". We propose a distinction between Philosophy and Criticism, and, building on a study of Lautman's sources and mathematical studies, we situate the main bulk of his work in the realm of Mathematical Criticism.

Keywords. Lautman, Hilbert, Herbrand, philosophy, criticism.

§ 0. — Introduction.

Analysis and synthesis always deserve to be thought of in counterpoint. In the case of Albert Lautman (1908-1944) it is interesting to *analyse* (disjoin) his work from the point of view of a "mathematical criticism" similar to literary, artistic, or musical criticism, before *synthesising* (conjoining) this approach with the more traditional views of "mathematical philosophy". *Section 1* offers a short discussion of the contrasts between "Philosophy of Mathematics" and "Criticism of Mathematics". *Section 2* looks at Lautman's sources, both quantitative and qualitative, and reveals his strong focus on mathematics at the expense of philosophy. *Section 3* looks at Lautman as a critic of the central mathematical masters of the period 1830-1930: Galois, Riemann, Poincaré, Hilbert. *Section 4* traces Lautman's references to his contemporaries, in particular to the work of Jacques Herbrand. Finally, *Section 5* studies the connections of the "critical Lautman", highlighted in the previous sections, with the better known "philosophical Lautman".

§ 1. — Philosophy disjointed from Critique.

We consider mathematics as a *thought*, i.e. a "mixture" [197][(1)]between techniques and ideas[(2)]. This thought can be looked at from several points of view, depending on how one responds to the simple adverbs *what, who, where, when, how, how much, why*. A mathematician will be particularly sensitive to what-how-many; a historian of mathematics will be sensitive to who-where-when; a classical philosopher will be sensitive to what-how-why; an analytic philosopher will be sensitive to what-how[(3)]. On the other hand, a *mathematical critic*, if such a being exists[(4)], will have to be attentive to *all* the what-why-where-when-how-how-why modalities, and will have to carry out a (*de*)*construction of mathematical thought on all the possible registers of creation*[(5)].

The analytic philosophy of "mathematics" has *reduced* its questions to the philosophy of logic and set theory, and focused on *flattened* versions of what (number) and how (deductive reasoning). With this reduction and concentration, analytic philosophers have constructed a precise and self-referential discourse, linguistically and logically powerful, but mathematically *empty*. The distancing from living and complex mathematics (space-form-structure, around geometry, topology, abstract algebra, the complex variable, analytic and algebraic number theory, functional analysis, differential equations, etc.) has been complete. With this, not only did the historical (who-where-when) and metaphysical (why) spectra of mathematics disappear, but the objects (what) and methods (how)

(1) References of the type [page] lead to the Vrin edition of Lautman's "Works" [Lautman 2006]. References of the type [name year, page] lead to the bibliography, indicated at the end of the article.

(2) As is well known, the notion of "mixture" comes from Lautman. We have extended this Lautmanian conception through the notion of *bundle*: mathematics comprises (at least) a "folded" space of techniques (definitions, axioms, proofs, examples) and an "unfolded" space of ideas (images, concepts, intuitions) which become "mixed" and project onto the techniques. *cf.*[Zalamea 2021a].

(3) Of course, these simple disjunctions only try to give a *broad orientation* for *communities of* scholars. Beyond any disjunctive analysis, there may be special cases of historians of mathematics who amply cover the full register (such as the exemplary case of Jeremy Gray), or philosophers of mathematics who also cover it (such as Gilles Châtelet).

(4) For a survey of the situation, see Zalamea's article in the first issue of *Annals of Mathematics and Philosophy*, [Zalamea 2021b].

(5) In this perspective, and going beyond the usual classifications, we could say of Gray and Châtelet that they are *mathematical critics* in the most accomplished sense.

were singularly impoverished. Perhaps this flattening was necessary in an attempt to achieve an (illusory) precision of language, but the fact is that this philosophical approach has left out the essential forces of mathematical creation[(6)].

The work of the *mathematical critic* must explicitly counter any reductive attempt, and deepen the *multidimensionality of* perspectives and interpretations. In this sense, a *healthy disjunction* with the philosopher must be made (before attempting a later conjunction, see *Section 5* below). The critic must, above all, (A) *know how to look at* mathematical works. Just as a literary critic who wants to study Proust, for example, must have read *À la Recherche* directly, a mathematical critic who wants to study Riemann, for example, must have read his *Doctoral Thesis* directly. The return to the sources, to the works, is a *sine qua non* for the critic, a condition often important for the historian, but often neglected by the philosopher. The what-when-where-when becomes really central for the critic: he cannot restrict himself to a secondary literature, far from the works, and develop abstract linguistic discourses, far from the mathematical structures. Secondly, having been confronted with the complex objects and processes of mathematical creation, the critic must (B) *accurately describe* the central ideas and associated techniques of the works submitted to his analysis (mathematical thinking as a "sheaf", answers to the how-how much). Finally, the critic must be able to (C) *explain the great forces* (whys) that sustain mathematical creation. In this sense, he goes beyond the mathematician, just as the literary critic goes beyond the writer, or the art critic goes beyond the artist.

Thus, thanks to a *full immersion in the broad adverbial panorama*, the critic (mathematical, literary, artistic, musical) will really approach the discipline studied (mathematics, literature, art, music). His or her work will be more *synthetic*, necessarily broader, since he or she will have to deal with a wider spectrum of perspectives. It will therefore be a complement to *analytical* work that is necessarily more limited. The *back and forth* between analysis and synthesis, between locality and globality[(7)], between language and vision, will have many advantages for a more *faithful* understanding of mathematics.

(6) The analyses of mathematical creativity by the French school — [Poincaré 1908], [Hadamard 1943], [Grothendieck 1985-86] — are still fundamental today.

(7) This is one of the recurring themes in Lautman's work, *cf.*[Lautman 2010].

§ 2. — Lautman's sources.

The point (A) mentioned above — looking at the works — is imperative for the mathematical critic. *Working on* mathematical *work* is one of his fundamental labours[(8)]. Lautman's mathematical knowledge has always been appreciated[(9)], and it is easy to look in detail at (*i*) his preparatory studies and readings (*source* study), and (*ii*) their use (*reference* study). Lautman's sources are multiple and complex[(10)]. On the one hand, his mathematical sources appeal to *treatises that are* now considered *classical*: mathematical logic (Hilbert & Ackermann 1928, Hilbert & Bernays 1939), number theory (Hecke 1923, Hurwitz & Courant 1925, Ingham 1932, Herbrand 1936), abstract algebra (van der Waerden 1931, Glivenko 1938), differential equations (Bieberbach 1923, Kähler 1934), complex variables (Picard 1896, Osgood 1907, Weyl 1913, Montel 1927, Nevanlinna 1936), functional analysis (Fréchet 1928), topology (Lefschetz 1930, Seifert & Threlfall 1934, Alexandrov & Hopf 1935), physics (Weyl 1928, Broglie 1932, Hilbert & Courant 1937, Julia 1938). The *dates* are remarkable: except for three treatises on the complex variable, *all* the others were published in the 1920s-1930s, highlighted as crucial by Dieudonné. On the other hand, Lautman is very attentive to *research papers* of his time: Hilbert 1923, Cartan 1924, Gödel 1930 & 1931, Herbrand 1931, Pontriagin 1931, Hopf 1932, Tarski 1935, Birkhoff & von Neumann 1936, Gentzen 1936, Weil 1938. Lautman also attended the *Séminaire de Mathématiques* of the Institut Henri Poincaré and benefited from brilliant lectures by Possel 1934-35, von Neumann 1934-35, Weil 1934-35, Cartan 1937. The doctoral theses of his friends Jacques Herbrand (1930) and Claude Chevalley (1934) are included in his work. References to

(8) Simone Weil has always defended the crucial importance of *work* in human experience, *cf.* [Labbé 2018]. For views on Simone's *mathematical eye*, *cf.* [Lafforgue 2014], [Zalamea 2022].

(9) According to Dieudonné's well-known phrase, "in contact with his comrades and friends Jacques Herbrand and Claude Chevalley (two of the most original minds of the century), (Lautman) had acquired a much broader and more precise view of the mathematics of the 1920s and 1930s than most of the mathematicians of his generation, who were often narrowly specialised; I can testify to this in my own case" [35]. Curiously, Yvon Gauthier reproaches him for this extension of his views: "his mathematical culture was broad, too broad perhaps not to remain superficial in many cases" [Marquis 2010, 159]. There is no reason for this rebuke: extension, precision and depth go hand in hand in Lautman's work.

(10) For an exhaustive study, see my "Noticia sobre las fuentes de Lautman", *cf.* [Lautman 2011, 543-558].

the *Mathematische Annalen* and *L'Enseignement Mathématique* show his desire to be well informed about current mathematical work.

From the point of view of (*ii*) *the* use of sources, it might be appropriate to describe Lautman's work as a sustained commentary, both analytical and synthetic, on Hilbert's profound work. Hilbert is cited 25 times as a source and mentioned in 47 pages of Lautman's body of work[11], both around his work in functional analysis, and in number theory and logic, where Herbrand's influence will have been decisive. From the quantitative point of view, the attention towards Hilbert is thus very sustained, but this attention is also very fundamental qualitatively: Hilbert's space is for Lautman the *sine qua non* example of a mixture, between continuous and discrete, local and global, finite and infinite. The second most cited mathematician is Riemann (1 reference as source, 38 pages of mentions). The other most studied authors are Élie Cartan (27 pages), Henri Poincaré (25 pages) and Hermann Weyl (23 pages), whose importance may have been pointed out to Lautman by Ehresmann, who studied with Weyl (1932-1934) and completed his thesis under Cartan (1934). Thus, under the aegis of Riemann-Hilbert-Weyl, as opposed to the French Galois-Poincaré-Cartan school, we see how Lautman's enormous mathematical culture is constituted, and how he moves towards a kind of *mixed structuralism*, very close to the mathematical boom of the 1920s and 1930s, which will become the true mark of his philosophical approach.

Indeed, far from an ingenuous "Platonism", which has poorly oriented the reception of Lautman's work, it is on a *complex combination of global structuralism and local mixtures* — prefiguration of the theory of sheaves, very clear from his "Rapport Bouglé" (1935), *cf.*[Lautman 2010] — that his original vision of mathematics is based. With Lautman, as with Cavaillès, effective (local) mathematics and conceptual (global) reflection go hand in hand, and it is precisely the evolving structures that allow the passage between the concrete and the abstract. It is enough to look at (*i*) his sources and (*ii*) his philosophical uses, to realise that Lautman can be much better understood as a *mathematical critic* than as a mathematical philosopher. In the following list, next to each philosopher, the first number indicates the number of references as source and the second number indicates the number of pages where the author is

[11] The source and reference counts here refer to the Spanish edition of Lautman's "Works" [Lautman 2011], which is more complete than the French editions (Hermann, PUF, Vrin).

studied: Plato (7 — 10), Descartes (3 — 3), Leibniz (3 — 6), Kant (3 — 9), Carnap (8 — 17), Wittgenstein (1 — 6), Heidegger (6 — 13), if we go through the most cited philosophers. The *myth of* the "Platonist Lautman" invented by the secondary literature is therefore immediately destroyed if we look closely at his writings. Not only are his philosophical perspectives *multiple*, far from being centrally Platonic, but they are far *fewer*, both in quantity and quality, compared with his mathematical mentions.

§ 3. — The critique of the Masters.

The task (B) for the mathematical critic mentioned in *Section 1* — to describe techniques and ideas with precision — concentrates the best of Lautman. His vision of the "proper movement of a mathematical theory" [66], of the "genesis of effective mathematical theories" [237], acquires great accuracy in his writings. Lautman deals with an enormous spectrum of mathematical constructions, as shown by a short list(12) of the scholars he studies: Bernays, de Broglie, Cartan (Elie), Dirichlet, Einstein, Euler, Galois, Gödel, Herbrand, Hilbert, Hopf, Klein, Lebesgue, von Neumann, Pfaff, Poincaré, Riemann, Schrödinger, Weierstrass, Weyl. In particular, there are magnificent descriptions of the major techniques and ideas introduced by the Masters of modern mathematics in the period 1830-1930: Galois, Riemann, Poincaré, Hilbert.

Galois' work is approached as an example of "the ascent to the absolute" [165]. According to Lautman, "the interest of the logical scheme of Galois' theory is considerable" [167], expressing the jump "from an imperfect basic domain" to "the existence of an extension where this imperfection has disappeared" [167-168]. The association of various numbers and measures at each stage of the ascent between two bodies then fulfils a *structural completion* fundamental to the Lautmanian vision: effective mathematics embodies a movement of *abstract intermediate mixtures*(13) which gives the discipline its deepest raison d'être. If the extensions of fields and its Galois groups are very useful to grasp the *variations*

(12) This list includes, in alphabetical order, mathematicians (and physicists) mentioned *on at least seven pages* in his work (reference to appearances in the complete Spanish edition [Lautman 2011]).

(13) "Study of possible modes of organisation of elements of indifferent nature", *cf.* [Lautman 2011, 378].

and invariances of mathematical thought, the meromorphic transformations and the representation theorems of Riemann around its multiple surfaces become for Lautman the perfect example to unveil the dynamic richness of modern mathematics. Indeed, the study of the uniformisation of Riemann surfaces in Lautman's Main Thesis [174-178] is a true masterpiece of exposition and understanding. The young "philosopher" becomes at this point a formidable *critic*, when he explains how "it is still a question of eliminating the imperfections of certain mathematical beings by passing from what they primitively are to an ideal of absolute simplicity whose existence is implied in the very entanglements of their structure" [174]. *The existence by decomposition and recomposition of the structure* (point (C) synthetic, explanatory of the critical labour) emerges as a result of long and precise preliminary analyses of the effective mathematics (point (B) of its descriptive labour). The analysis of multiformities, ramifications, overlaps of functions of complex variable, exemplified around algebraic functions and n-th roots [175], is linked to the topological characteristics of the associated Riemann surfaces [176], before arriving at the conformal representation theorems (Riemann, Poincaré, Hilbert, Koebe) of a simply connected Riemann surface "either on the whole complex sphere, or on this sphere from which a point is removed, or on the unit circle of the complex plane" [177]. Dazzled "by the immensity of the new horizons" [177], the critic Lautman succeeds in his triple task of *opening up mathematical intelligence*: (A) to look, (B) to describe, (C) to explain major turning points in thought.

Lautman studies — looks at, describes, explains — several central aspects of Poincaré's work: duality theorems in algebraic topology [160-163], topological methods in differential equations [215-218], hyperbolic metrics and the uniformity theorem of Riemann surfaces [101-102]. In addition to constant references to the links between topology and algebra invented by Poincaré, Lautman notes the interest of particular theorems in understanding general problems: the "properties of internal structure" opposed (and conjoined) with the "extrinsic properties of situation" [163], the "conditions of existence of fixed points" which depend as much on the "structure of the basic domain" as on the "nature of the internal transformation" [217], "the primacy of geometric synthesis over that of 'numerical' analysis" [105]. Here we see the critic in all his technical strength, capable of unveiling vast panoramas from close studies of mathematical works.

Around Hilbert, Lautman highlights his contributions to infinitary forms and functional equations [93-95], metamathematics and proof theory [129-131], the theory of class fields [168-169], infinity and the universal counterexample logical operator τ [180-181], Hilbert space [200-207], the Dirichlet problem [213-214], quantum mechanics [286]. Thus, Lautman traverses the vast spectrum of Hilbertian mathematics and really immerses himself in the *task of the critic*, attentive first (A-B) to the delicate technical expressions (fundamental forms of mathematical creativity) and then (C) to the great forces that unfold in it: "unification of mathematical disciplines" [93], "duality of planes (...) between formalised mathematics and the mathematical sciences" [94], "the development of the mathematical sciences" [95]) "between formalised mathematics and the metamathematical study of this formalism" [130], "solidarity of structure between the elements of a whole and the whole to which they belong" [169], legitimisation in the same way "to speak of the existence of the object as of the existence of the point at infinity in geometry, imaginary numbers, or ideal elements of a number field" [181], recognition of Hilbert space (always named in the singular, before the general axiomatisation of von Neumann) as "mixed (..) homogeneous to the continuous by the nature and topology of its elements, and to the discontinuous by its structural decompositions" [207], recognition of the Hilbert proof of Dirichlet's principle as a "mixed intermediate between the structure of the domain and the existence of the function" [213], the place of Hilbert space in the study of processes of evolution and propagation of physical quantities in mechanics [286]. *A continuous pendulum back and forth between descriptive analysis* (*A-B*) *and explanatory synthesis* (C) is one of the critic's own and characteristic strengths, beyond philosophical positions that would rather assume a firm categorical perspective.

§ 4. — **The criticism of contemporaries.**

The list of mathematicians of his time studied by Lautman is very indicative of his desire to be at the forefront of research: Ahlfors (2), Alexander (5), Alexandrov (5), Bernays (10), Bieberbach (5), Birkhoff (2), Caratheodory (2), Chevalley (5), Courant (5), Ehresmann (3), Fréchet (6), Glivenko (2), Gödel (9), Hasse (2), Hecke (6), Herbrand (17), Hopf (12), Lefschetz (3), Lukasiewicz

(2), von Neumann (7), Takagi (4), Tarski (3), van der Waerden (5), Weil (2), Weyl (24)[14]. Two names in the list are particularly important for Lautman: Hermann Weyl and Jacques Herbrand. Both cases are strong examples of *transitions* between the Hilbert school and the contemporary mathematics where Lautman is located. On the one hand, Weyl contributes to the understanding of *space* (via his monograph on Riemann surfaces (1913), quoted 5 times by Lautman) and *geometric* magnitudes (via the continuous-discrete dimensional analysis from Hilbert spaces). On the other hand, Herbrand contributes to the understanding of *number* (via his monograph on the theory of number fields (1936), cited 1 time) and, above all, of *arithmetic* magnitudes linked to his proof theory (via his Thesis (1930), cited 5 times). Thus, taking advantage of Weyl and Herbrand, Lautman obtains a good vision of the *space-number* dialectic, always crucial for a full understanding of mathematical thought.

The Weyl case is linked to central themes of modern mathematics (1830-1930): intuitionism [42], group theory [48, 83, 85-87, 117, 143-145, 193-194], Riemann surfaces [85, 101-102, 135, 171-178, 188], topology [113, 117, 121, 154, 188], functional analysis [154, 206]. The *critic* Lautman draws on (A) Weyl's work to (B) describe geometric techniques in counterpoint to abstract algebra, and to deduce (C) a deep understanding of the modern "war" between space (the "angel of topology", according to Weyl) and number (the "devil of algebra"). Lautman's *very detailed descriptions* of Weyl's work on group theory and Riemann surfaces help us to really *grasp* the techniques involved. This work (B) of the critic — often misunderstood by mathematicians, philosophers, historians — is very useful to come to a better understanding of mathematical thinking. Often, the mathematician himself is unable to (B) describe his creations, which the historian or philosopher simply (A) does not look at. The critic, *forced* to join the analytical steps (A) and (B) to arrive at the synthetic explanation (C), turns the whole situation around, and develops a task *complementary* to other approaches to mathematics.

The Herbrand case is one of the gems of Lautman's *analysis-synthesis.* Very close to his friend, Lautman remarkably explains the construction of *Herbrand's fields* in his theory of demonstration, following Galois' fields. The two precocious geniuses, Galois (19

[14] The numbers in brackets correspond to the *number of pages* in the Spanish edition [Lautman 2011] where mathematicians are mentioned.

years old) and Herbrand (22 years old), come together under the vision of the young critic (30 years old). Herbrand, a disciple of Hilbert [43], constructed his method of fields to ensure in certain cases the non-contradiction of a theory [45-46]; the theory cannot be too "high" (*e.g.* cover analysis), but works for *bounded strata of arithmetic*; what results essential is the construction of an "intermediate schematic, that of individuals and fields considered not so much for themselves as for the infinite consequences that the finite calculations operated through them allow" [46]. Lautman presents "Herbrand's theorem" as a profound structural-syntactic connection: "the existence 'in extension' of an infinite field where *non-P* is feasible is equivalent to the structural fact that P is not part of the set of provable propositions of the theory" [186]. It is "a pure case of solidarity between a set of formal operations defined by a system of axioms and the existence of a field where these operations are realizable" [180], which responds to a fundamental creative conception: "the essence of a form being realized within a matter that it would create, the essence of a matter giving rise to the forms that its structure draws" [186]. The construction of Herbrand's fields is described in detail (indices, functions, iterations, growth) [198-200] as a *mixture par excellence*(15) between mathematical infinitude (individual values) and metamathematical finitude (recursive functions before the letter). Thus, (A) immersing himself in an arduous doctoral thesis, (B) describing the main mechanisms emerging from it, and (C) reflecting on the method, combinatorics and structuring of the theory of proof proposed by Herbrand, Lautman accomplishes the supreme and complex task of mathematical criticism: to *see in detail and in depth, in order to make us see better*.

§ 5. — Criticism in conjunction with Philosophy.

(15) "Intermediaries between signs and their values, these fields are, on the one hand, homogeneous to the finite discontinuity of signs since to a sign of a variable corresponds only one value a_i and, on the other hand, they symbolise an infinity of mathematical values since the letter a_i represents any possible mathematical value of the variable y when it intervenes in the particular form. A mediation is thus carried out by these fields from the finite to the infinite, which makes it possible in the cases treated by Herbrand to dominate the infinite and such is the role that we will recognise in the mixed fields (...)". [200].

The work of the philosopher extends to a fourth level of (D) conceptual reflection, speculative discourse and dialogue with the history of philosophy. Lautman is a magnificent example of the *conjunction* between the critic (A-C) and the philosopher (D). Some of Lautman's strong conceptions (notions, Ideas, mixtures, dynamic Platonism, effective mathematics) are situated in this more philosophical level, but always *building on his profound critical work*. However, the situation is far from being the same with other "philosophers of mathematics". Many divergent situations can occur: (*i*) weak or no mathematical level (A-B), and strong (C-D) but restricted to logic, (*ii*) strong (A-B-C) mathematics, devoid of (D) philosophical considerations, (*iii*) strong (B-C-D), far from analyses of the original mathematical works, (*iv*) strong (A-B-C-D)

It is as a *critical philosopher* — arguably the major one of the 20th century — that Lautman deserves to be evaluated. And it is probably because of this difference with standard philosophy that the work of the young French thinker, *critically* focused on the *mathematics of strata* (*A-B*), has not really been understood by the so-called "philosophy of mathematics", essentially Anglo-Saxon, turned entirely to the *logic of strata* (*C-D*). Indeed, to make a philosophical critique of mathematics is a completely different task from making a philosophy of logic. It is *thanks to* his critical and mathematical vision (A-B-C) that Lautman can propose his most original philosophical ideas: (1) the construction of *Ideas* as partial resolutions of *notions* (*e.g.,* the Cantorian Idea of the continuum as a resolution of the notions of completeness and discreteness, but also, reciprocally, the Brouwerian Idea of the continuum as an *inverse* resolution to the Cantorian one, and so on), (2) the development of *mixtures* as a central force of mathematical creativity (e.g., Galois theory, Riemann uniform surfaces, Hilbert spaces, Herbrand fields, etc.), and (3) the understanding of a *dynamic Platonism as an iterated back-and-forth between various strata of* understanding (*e.g.*, particular *effective mathematics* in contraposition with general Ideas).

A rich and concrete mathematical material (A), a close analysis of this material (B), a deep synthetic understanding of the established forces (C), and a *broad* philosophical reflection (D) based on these labours, constitute the specificity and originality of Lautman's work. We should hope that the *young French philosopher-critic of mathematics* will be set as an example of this "difficult thinking", hailed by Bachelard, which tries to reflect and understand in depth mathematical invention.

§ — Bibliography.

Works by Albert Lautman.

[Lautman 1977] *Essai sur l'unité des mathématiques et divers écrits* (ed. Maurice Loi), Paris: PUF (10/18), 1977.

[Lautman 2006] *Les mathématiques, les idées et le réel physique* (ed. Jacques Lautman), Paris: Vrin, 2006.

[Lautman 2010] "Report on the philosophical work undertaken by Mr. Lautman" (1935), in [Marquis 2010], pp. 9-15.

[Lautman 2011] *Ensayos sobre la dialéctica, estructura y unidad de las matemáticas modernas* (trans. & ed. Fernando Zalamea), Bogotá: Universidad Nacional de Colombia, 2011.

Studies.

[Hadamard 1943] **Jacques Hadamard**, *Essai sur la psychologie de l'invention dans le domaine mathématique*, New York Lectures, 1943 (English publication, New York: Princeton University Press, 1945; expanded French translation, Paris: Albert Blanchard, 1959; reprint, Paris: Bordas, 1975).

[Grothendieck 1985-86] **Alexander Grothendieck**, *Récoltes et semailles*, manuscript, 1252 pp.

[Labbé 2018] **Mickaël Labbé**, *La notion de travail chez Simone Weil*, Paris: Demopolis, 2018.

[Lafforgue 2014] **Laurent Lafforgue**, "Simone Weil et la mathématique", in: E. Gabellieri & F. L'Yvonnet (eds.), *Simone Weil — Cahiers de L'Herne*, Paris: Éditions de L'Herne, 2014, pp. 126-137.

[Marquis 2010] **Jean-Pierre Marquis** (ed.), *Albert Lautman, philosophe des mathématiques, Philosophiques* 37(1) 2010, Montréal.

[Poincaré 1908] **Henri Poincaré**, *L'invention mathématique*, Paris: Institut Général Psychologique, 1908.

[Zalamea 2019a] **Fernando Zalamea**, *Philosophie synthétique de la mathématique contemporaine* (trans. Charles Alunni), Paris: Hermann, 2019.

[Zalamea 2019b] —, *Grothendieck. Una guía a la obra matemática y filosófica*, Bogotá: Universidad Nacional de Colombia, 2019.

[Zalamea 2021a] —, *Modelos en haces para el pensamiento matemático*, Bogotá: Universidad Nacional de Colombia, 2021.

[Zalamea 2021b] —, "Towards Mathematical Criticism", *Annals of Mathematics and Philosophy* 1 (2021), in progress.

[Zalamea 2022] —, "Géométrie, topologie, Riemann, et les nuances vivantes de la pensée mathématique chez Bachelard (avec un contrepoint autour de Simone Weil)", *Bachelard Studies* (2022), in press.

Fernando Zalamea, Departamento de Matemáticas, Universidad Nacional de Colombia, Sede Bogotá.

⋆ ⋆ ⋆