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The mathematical philosophy of Maximilien Winter (1871 – 1935)

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Abstract. Maximilien Winter was at the heart of the French epistemology, shaping it by creating the *Revue de métaphysique et de morale* along with Brunschvicg, Couturat, Halévy and Léon. Now forgotten, we describe his singular philosophy through a study of his abundant bibliography in the *Revue*.

Winter's philosophy stands in direct opposition to the metaphysics, positivism and logicism then prevailing. He rehabilitated instead a critical and historical epistemology of the ideas and methods. According to Winter, the role of philosophy is to clarify and organize the principles of science, through a technical analysis of the scientific texts and the evolution of ideas.

§ 1. — Introduction: The purpose of the philosophy of science.

Overview of his life, influences and orientations. Maximilien Winter (1871 – 1935) studied at Lycée Condorcet in Paris, where he became friend with Léon Brunschvicg, Louis Couturat, Élie Halévy and Xavier Léon. This relationship led to the founding of the *Revue de métaphysique et de morale* in 1893, whose stated aim was to provide philosophy with an effective way to defend its status against the overwhelming rise of the sciences, while opposing positivist speculation [25][(1)].

[(1)] The provided reference focuses on the lives of Léon Brunschvicg, Élie Halévy and Xavier Léon. Beyond this survey, the only information we have been able to gather on Maximilien Winter comes from his own writings — nearly forty years

Winter studied law and became a lawyer, giving ground to some of his future work on legal controversies and their relations with mathematical logic ([4] and [6]). He had a passion for mathematics and philosophy of sciences, and published various articles on epistemology in the *Revue*. He has been influenced by his teacher Alphonse Darlu (see [23]), who tainted his philosophy with Platonic and neo-Kantian ideas. Winter defends a systematic philosophy of science, based on a dialectic whose rigor is modeled on logic and guarantees the objectivity of the arguments.

Because of these aspects, Winter is far from supporting the re-establishment of metaphysical and moral philosophy, ubiquitous in the editorial line of the *Revue*, which for him is, on the contrary, a pitfall to be avoided. His strong rejection of metaphysics or any argument based on absolute or theological postulates depict his interest in Auguste Comte and Hegel ([22]). However the philosophers of the *Revue* remain united by the acceptance of a central role of science in philosophy, although Winter does not share the visions of a superiority of philosophy over science — he gives on the contrary an essential place to the study of scientific works themselves. The young philosopher found his place in the *Revue*, which, unlike its counterparts, refrained from promoting any particular doctrine, and wished to become a relatively free forum for philosophy in France. Winter's affinity with the other founders of the *Revue* undoubtedly favored his participation.

Winter altogether seeks to rehabilitate the role of philosophy as a reflection on the principles of science against the positivist approach, to reintroduce scientific and technical practice against the metaphysical approach, tracing a path between the metaphysical orientations of the *Revue* and the positivism it faces.

An interest in modern and scientific issues. From his earliest writings, Winter devoted himself to an in-depth study of scientific theories, recounting the evolution of ideas, and focusing on the scientists' own judgments about their discipline and the fruitfulness of their methods. This proximity, which he defends throughout his work, between philosophy and science, between thought and

of articles published exclusively in the *Revue* — along with the three articles [23], [24] and [25] which deal essentially with the history and philosophy of the *Revue*, as well as the obituary which is dedicated to Winter. We regret that we did not have the time to study in depth the philosopher's correspondence, which is preserved in the Victor Cousin library at the Sorbonne.

its object, will be the common ground of his studies. The constant to-ing and fro-ing between the two worlds would remain the driving force behind his reflections, set up as the only legitimate way to unify science and philosophy, in line with a certain intuitionism dear to Kant and Poincaré.

Winter was particularly interested in the scientific upheavals of his time. Although the scientific subjects of his work appear eclectic (analysis, arithmetic, mechanics, quantum physics, relativity, biology, law), they are distinguished by a certain topicality. His studies focus on the contributions of new theories, both within the sciences and insofar as they call into question many philosophical positions: Frege's logic, Cantor's set theory, Hilbert's axiomatization, Heisenberg's quantum physics, Hadamard's analysis, and Einstein and De Sitter's relativity.

His research focuses on the axiomatization of science, seen through critical and rationalist glasses. The problem was in vogue at that time when logical theories were blossoming, and while originality was not a feature of his subject, his position was less common. Like Comte, Winter not only categorically rejects the metaphysical reasoning of the philosophers and mathematicians of the time, but, as Kant's heir, is careful not to sink on the other hand into the absolute logicism that Winter would later say was only a blinding moment for mathematicians and philosophers. On the contrary, his task is to reveal a new role for philosophy and history in the development of science.

An organizing and clarifying philosophy. Scientific and rationalist in spirit, Winter sets out to clarify the possible methods for studying the principles of science. He depicts the philosopher's mission as follows: "Philosophical thought is neither demonstrative nor merely critical; it is organizing, or more precisely systematic, and aims essentially, if not to define [the systematic principles of science] in the scientific sense of the word, at least to determine them sufficiently for the organization of our ideas" ([2], pp. 615-616). This approximation in determination, marking the mourning of absolute definitions, is a first sign of a philosophy that will remain deeply rooted in the practice of science in the making, and to which an organizing philosophy, however incomplete, can contribute. His *new conception of the philosophy of science* is spelled out in his seminal article [2]: "All sciences must strive to realize the type of certainty: A is A. But reality offers us only heterogeneous

facts; the problem that science will have to solve will therefore be to find relations of identity between heterogeneous facts. But [...] it cannot be directly, only by means of detours, that the mind can establish relations of identity between heterogeneous facts. The task of the philosophy of science will therefore be to determine these detours of the human mind." ([2], p. 607) The role of the Winterian philosopher thus appears above all as a quest to clarify the principles underlying the sciences.

This does not mean embracing a philosophical relativism in which every object is bathed in a specific philosophy. On the contrary, "we admit [...] that there is room outside mathematics for different orders of things, for distinct methods: but we don't give up on establishing that the different principles on which methods are based are not arbitrary conceptions, but rational principles. The philosopher's task will not be to catalog the subterfuges of the human mind, but to rationally determine the principles on which the various methods are based, and to show their systematic sequence." ([2], p. 613) Winter's quest, then, is not merely to uncover principles existing in practice, but to identify their consistence and, if not to establish a system, at least to clarify their relationships.

Such a task is multifaceted, and throughout Winter's work it resembles an investigation into the philosophical postures governing science, followed by a discussion of the value of these principles, in order to avoid certain biases and identify new interpretations. More precisely, "there is an important role for the philosopher to play in science, without confusing it with the work of the scientist. [...] A small number of new attempts are indeed interesting discoveries, most are irrelevant and many are false. It is important that these attempts are sorted out, that a critic judges the theories as they are born. This criticism is to the sciences what dramatic criticism is to plays [...] and will be better done by a philosopher with a solid scientific education, than by a learned inventor [...] There is a second object of study for the philosopher, and that is the philosophical history of science, [...] history conceived as the very genesis of scientific theories, where the filiation of fundamental ideas would be established [[7], p. 326]. To sum up: "criticism of science, philosophical history of scientific theories, contribution to the formation of scientific methods: these are, in our view, the three forms in which philosophical thought manifests itself" ([7],

p. 327). Winter's work instantiates a philosophy deeply rooted in science and its history.

A precise analysis of philosophical postures on science. Winter goes beyond the search for unification of the sciences, and instead places primary importance on the study of the variety of methods and principles. He points out that the syllogism, mathematical equality, the method of geometric superposition, experiments in physics and legal reasoning are all applications of this principle [of identity], yet the relationships between concepts, the equality of quantities, the coincidence of figures, the experimental method of physics and discussions of civil law are all very different methods. Consequently, to abstract from the sciences everything that cannot be reduced to the pure form of identity is to examine only one facet of the philosophical problem, to identify the element common to all the sciences; but philosophy must also determine the constituent and specific differences between principles and methods." ([2], p. 610) This eclecticism of subjects, reflecting a diversity that should not be masked, became and remained fundamental to his work.

Winter identifies three dominant philosophies and approaches to thinking about science and the search for its foundations, which he discusses and confronts throughout his work, and against which he draws his philosophy of science. We summarize these stances and dedicate this article to clarifying Winters' positions and arguments.

The first posture is metaphysics, a method of reflection that is thousands of years old and never exhausted, whose impact and ubiquity he notes as much in philosophy as in scientific reflection and in the arguments of scientists themselves. Winter calls metaphysics "any theory of perception and knowledge, or any system of explanation of the ultimate constitution of matter, which employs neither the mathematical method nor the experimental method of the physical sciences, and which will not expand the domain of positive science." ([3], pp. 509-591) This metaphysics is alien to experiments, and is for Winter rather a philosophy of first principles, rising into a theology above scientific reasoning and objects, but thereby outside science.

Positivism is a movement that emerged with Auguste Comte in the 19th century, establishing confrontation with facts and experimentation as the ultimate criterion for scientific judgment. In

the positivist view, science aims to describe precisely the relationships observed in facts, in opposition to metaphysical speculation. It is thus of the same nature as science, using the same tools of exploration, and aiming to order the findings into laws of nature. Although Winter adopts a philosophy closer to science, refusing to dissociate himself from experiments and observations, he nevertheless gives the philosopher the role of thinking about the principles of science, clarifying and organizing them, going beyond mere positivist observation.

Finally, logic has progressed throughout history, becoming almost totally dominant at the beginning of the 20th century, after a dazzling conquest of most mathematical and philosophical circles. Its development within mathematics spread to other scientific fields and philosophical schools. This new logicism, driven by the axiomatic theories of Frege and Cantor, as well as the formalist efforts of the Hilbert school, postulates that formalization is the way to embrace the rules of truth and mechanize thought. This leads Winter to ask: "Shouldn't we assume that a logic, conceived in the same spirit, will one day, as it develops, absorb the philosophy of all the positive sciences?" ([3], p. 591) This enthusiasm for a formal, axiomatic science is very much present in Winter's work, but with restraint and restriction, considering logic as a tool for clarification and control rather than as a unique and sufficient method for founding the sciences.

Winter rejects each of these three conceptions of the philosophy of science, and sets himself the task of finding a middle ground between these radical positions. "The question is whether scientific judgment is purely abstract, general and analytic (in which case the very movement of scientific thought would tend to eliminate all empirical content), or whether it is synthetic (i.e. whether it concretely unites the law with a particular content that cannot be separated from it). Perhaps we should say that scientific judgment is neither analytic nor synthetic, but that it lies somewhere between the two. It is this latter thesis that we shall attempt to demonstrate." ([1], p. 171). This declaration of intent, from his very first study, would remain his constant philosophical program.

Winter describes his philosophical approach as the "critical history" of ideas and methods. The scope of this approach is described by Winter as huge and not taken into consideration enough. It is about establishing a detailed understanding and close examination of scientific paradigms and objects, and judging them through the

glasses of a historical epistemology. Winter claims philosophy's right to address technical issues, defending a philosophical way of thinking shaped by "[the] critique of science, [the] philosophical history of scientific theories, [and the] contribution to the formation of general methods and theories" ([7], p. 327). It is thus an a posteriori method, essentially based on a justification derived from experience, practice, understanding of the ideas of science and their fruitfulness.

All Winter's work, his precise studies of scientific theories and his reflections on science, are geared towards comparing these methods, which are often combined in practice, in order to clarify their underlying values and principles. Winter restores a strong place to technical scientific studies, without limiting philosophy to a positivist attitude of description, and gives full scope to the reading of the history of science as the only way of thinking about scientific development by drawing out a natural movement of principles (see part 2). From the methods he outlines, he banishes metaphysics as an epistemology (see part 3), and severely restricts the philosophical and absolute framework of logicism without denying its primordial importance in reasoning (see part 4), paving the way for a new philosophy.

§ 2. — A philosophy close to science.

A critical philosophy for fruitful principles. The constant in Winter's philosophy is to maintain a close relationship with the object of science. This object is external to thought, and science only develops models of it, but it remains the criterion for judging principles and results. His stance could thus be described as scientific criticism, in line with a certain positivism. "The criticism keeps the experimental shape of the content to avoid that metaphysical principles, mere condition of thought, are transformed into dialectic ideas" ([2], p. 619). Experience is not, however, the absolute criterion, as positivism claims: "philosophical positivism, which condemns human thought to becoming more and more limited, is belied by the very progress of science [...] Only the rational ideal which affirms the supremacy of calculation and the possible resolution of the universe into mathematical principles, conforms to the

spirit of science, and can constitute a principle of enthusiasm and excitement in the search for scientific truth" ([3], p. 619).

The philosopher's role is not, therefore, to confine himself to the observation of facts, as positivism would claim. While it is not his role to interfere with the scientist's work in the application of methods and techniques, whose virtuosity he cannot match, "it is in the determination of new principles that the philosophical spirit intervenes" ([8], p. 912) and guides the scientist, helping to identify research directions, if not to shed new light on results and methods. Thus, "The overall exposition of the elementary principles of a fundamental science like Analysis, when it is made by a great mathematician, always has a philosophical scope" ([21], p. 607), marked by a critical retrospect on ideas. By combating a fragmented presentation of mathematics, which only serves to conceal its interest and distort its harmony, Hadamard is the model of the Winterian philosopher-mathematician, "striving to link solutions, by rational methods, to the fundamental principles" ([21], p. 608) that he extracts from the many problems and aspects of mathematics to illuminate its unity.

Critical philosophy is not just an abstract posture, but also the best way to judge the progress of science. Indeed, the real significance of new facts needs to be identified philosophically, by drawing out their fruitful principles independently of the emotions aroused within the field. He notes that, "when a new scientific theory appears, it sometimes determines our general ideas in a way that seems to shake the very foundations of science. Then, as time goes by, things settle down and sort themselves out. We realize that the scientific universe has not been turned upside down, but that a special truth has simply been added to the truths already known" ([14], p. 268). The position of the philosopher as thinker of the sciences thus guarantees a certain objectivity to scientific reflection, avoiding dazzlement and overly vivid conclusions. The exteriority of philosophical reflection frees thinking about science from its internal cultural conditioning. This presence is all the more necessary because, given the perpetual accumulation of new ideas and results in science, "it is important [...] that a critic judges theories at their birth. This criticism [...] will be better done by the philosopher with a solid scientific education, than by a learned inventor" ([7], p. 326). The importance of technical and scientific ability in understanding the essence of scientific work with sufficient finesse to be able to discuss it stands in stark contrast to many philosophical practices, which remain remote from technical details. Winter endorses this role throughout his work, analyzing

highly technical details, synthesizing principles and judging many of the scientific developments of his time.

This reflection on principles should not, however, engulf science, but merely enlighten it to a necessarily limited extent: "whenever a problem can be solved with existing methods, by more or less complicated artifices of calculation, but without new principles, philosophy has absolutely nothing to do in this work [...] let us add that results of capital importance for science have been obtained in this way" ([8], p. 912). We must thus "determine [the principles] solely for the scientific use for which they are intended [...], highlight only those principles that are necessary for positive science, and seek nothing else in this work on elementary notions, than to give a sufficient determination of them for scientific use" ([8], p. 913). This new point of view, this reflexive critique, must therefore be carried out by identifying the principles that really have their utility, by seeking a "determination of them that is sufficient for scientific use" ([8], p. 913), affirming the criterion of utility as the philosopher's judge.

A historical method for ordering ideas. Winter's chosen approach to this critique is the historical epistemology, studying the evolution and arrangement of ideas and principles in the course of the evolution of science. This posture preserves the link to the discipline, while retaining external attachments so as to be able to judge ideas and postulates with hindsight. It also goes beyond a simple description of results, seeking to determine the central principles governing theories, their evolution and their relationships. "Taking our inspiration from Mach, and seeking to apply to mathematics the critical-historical method that the eminent thinker developed in his study of the formation of the principles of mechanics, we shall sometimes seek to show how the idea evolved, and sometimes characterize, outside of all historical considerations, the system of elementary notions in their logical sequence. This method is legitimate, because a completely objective history of the sciences is impossible: the only truly objective historical method would be the complete publication of everything that appears, a method whose absurdity is plain to see. Criticism will always intervene to discern the important works that must constitute the fundamental scientific theories." ([10], p. 495) This illustrates the historical-critical method used by Winter to achieve his aims of clarifying and organizing the principles and ideas of science. Aware of the

accumulation of scientific knowledge, "scientific criticism may have yet another interest. By seeking to highlight the fundamental ideas found in the most recent theories, it can contribute to the work of *elementarization* of notions that is necessary for the progress of science. Simplification is an ongoing process needed in parallel to the growth of knowledge." ([10], p. 496).

On the other hand, "we know that a certain implicit philosophy guides the scholar, if not when he seeks to solve clearly stated problems, at least in the choices of problems he addresses, and it is the axioms of this implicit philosophy that it would be interesting to uncover and criticize" ([14], p. 276). This remark constitutes, as we said, the keystone of Winter's philosophical edifice: a quest to clarify and organize the often implicit principles of the sciences. Such a philosophy does not claim to determine the ultimate principles of thought, but the scientist must gain from awareness of this underlying philosophy, complementing his technical virtuosity. Winter systematically shows the fruits of such historical study, for example in his study of the history of concepts and methods in number theory, where "most of the great ideas that transformed algebra and analysis [...] were also fruitful in arithmetic" ([7], p. 323). It is in this way that critical philosophy revives the unity of the world, fragmented in its study by science, by drawing out constant generalizations, as he shows on the example of function theory ([13]), where the advent and evolution of functional calculus, seemingly disordered and advancing in various directions, follow unifying principles: the philosopher's task has been to bring them to light. The greater scope and penetration conferred by philosophical exploration has been echoed historically by mathematicians such as Hadamard, Leibniz and Poincaré, whose scientific impact has been huge. The philosophical and critical study of the evolution of ideas and methods, for example, reveals the profound links between analysis and arithmetic, yet so distinct at first and in the inner practice of the scientist, thus revealing a larger and more complete whole formed by them, whose parts are interdependent ([7], p. 344). Another example is provided by Weyl's work, which "is above all an effort to logically adjust the principles of differential and kinematic geometry to the theory of relativity" ([16], p. 28). More generally, "the philosopher's task is to justify the work of science by showing that the principles on which science rests are not an incoherent set of arbitrary symbols, but form a rational organization" ([2], p. 620), made explicit by the philosopher. This

historical-critical approach thus appears fundamental, both to shed light on the evolution of mathematics and to contribute to the associated philosophical reflection, which "naturally emerges from the history of science itself" ([15], p. 666).

The usefulness of philosophy in science. Winter defends the role that philosophy can and should play in scientific research, and its usefulness within the sciences. He argues that "criticism will always intervene to discern the important work that must constitute fundamental scientific theories" ([10], p. 495) but "the work of criticism we are talking about is not only of purely philosophical interest, it can be of great use to science" ([4], p. 496), as we have already mentioned. Philosophy is thus placed at the service of science, a position legitimized by the observation that "the spontaneous development of philosophy in scientific circles is the best proof of its vitality and usefulness" ([8], p. 911). In fact, most of these philosophical works were written by leading scientists whose knowledge of their subject matter cannot be attributed to them.

Many examples of the usefulness of philosophy in science are highlighted by Winter. Dubois-Raymond and Cantor are the finest examples of the fruitfulness of philosophical method combined with science ([8], p. 918). Both drew on the lessons of philosophical reflection on the mathematical principles that guided them. "If there are discoveries in science that are due to skillful calculations, others to ingenious experimental devices, there are, on the contrary, some that have their origin in the meditative effort of a philosophical mind" ([7], p. 326). A close study of Leibniz's and Newton's work on the genesis of the infinitesimal calculus shows that "Leibniz's inferiority as a calculator was more than offset, in the creation of a method, by the breadth of his philosophical thought" ([7], p. 327), to the extent that some of the most essential advances in positive science have been made by philosophical thought ([7], p. 327). Another example is provided by the development of algebra described in [10]: although Lagrange failed to establish a general method for solving equations, based on those of Cardano and the Italian school in the case of small degrees, he nonetheless introduced the fundamental concept of a resolvent function and developed the notion of symmetries and permutations. These new ideas were key to Galois's later studies, as he succeeded in grasping the fundamental notion of a group associated with an equation and characterizing its resolubility. The

demonstration of these structures led to Abel's proof of the non-solvability of general equations of degree greater than 5, motivating a reform of the notion of the solution of an equation. This also gave rise to the means, so fruitful in analysis, of representing solutions as limits of successive approximations, through transcendental functions, integral forms, development in series, etc. It is also Klein's philosophical reflections that take the unification of geometries a step further ([10], p. 523). The enlightening and organizing role of Winterian philosophy is richly illustrated here, and judged through the eyes of the usefulness within the sciences.

Finally, scientists and philosophers play a complementary role in the establishment of science and the development of ideas, and "while the scientist is exclusively interested [in] the new and precise fact that has been added to the body of knowledge, the attention of the philosopher, more romantic by temperament, is drawn above all by the disturbance in our ideas" ([14], p. 268). "Applied to the positive sciences, philosophy, far from being, as the ignorant suppose, a distraction for idle minds, constitutes on the contrary the very principle of fermentation that excites minds too inclined to contract machine-like habits" ([8], p. 920). This posture "prevents scientific thought from crystallizing" ([14], p. 269), clearing the way for new ideas. But "neither can it be seen as a kind of universal speciousness, in which mathematical thought would be completely annihilated" ([5], p. 214). As well as being a unifying and organizing principle, philosophy of science also serves a useful function for scientists. Conversely, "there are mathematical problems, properly speaking, that raise far-reaching philosophical questions" ([5], p. 214).

§ 3. — Winter against metaphysics.

The rejection of metaphysics. Despite the rationalization of French and German schools of thought since the 18th century, notably with the philosophy of the Enlightenment, Kantian criticism and the advent of modern logic, metaphysics remains at the heart of philosophical developments. Metaphysical arguments are used in ways that are often implicit, sometimes unconscious, but always underhand. In this way, philosophies are often partially tinged with metaphysics, the rigor of reasoning sometimes giving

way to the intervention of a transcendent evidence that withdraws its legitimacy, in the sense of a scientific epistemology, from the reasoning as a whole. Such is the position of Dunan, who posits at the outset of his study that "science is the starting point of metaphysics" ([1], p. 167) and who derives from science a metaphysics that governs it, subjecting one to the other through a philosophical postulate rather than an argument respecting explicit methodologies or rules.

Many other scientists and philosophers adhere to such a metaphysic stance. Such is the case with Kant, whose transcendental logic Winter recognizes as valuable, but whose metaphysical aspect unfortunately takes him beyond science ([3] and [11]). Poincaré also rightly defends himself against drift and utopianism, without taking care to avoid the final metaphysical argument in favor of the intuition he defends ([9]): Winter emphasizes at the outset his intention to avoid this caveat by "distinguishing[ing] in problem[s] the methodological point of view and the metaphysical point of view" ([9], p. 921).

From Winter's very first article, the young philosopher tackles the transcendent aspect characteristic of metaphysical postulates. He illustrates this with the example of Charles Dunan's vitalist theory ([1], p. 169), and despite agreeing on the existence of a living unity that goes beyond the mere union of body parts, the only way to justify it, according to Winter, should be through thought and science, whereas Dunan, conversely, justifies thought through a vitalist metaphysical postulate. Metaphysics is presented by Winter in its aspect outside science, beyond its object, and therefore also alien to it. But "metaphysics is not a science, nor can it provide scientifically indisputable formulas" ([3], p. 595), and only science and deductive thinking are sufficiently precise to prevent reflections from being biased by an arbitrary and unfounded initial postulate, such as those of metaphysics. Thus, at the risk of borrowing from logicism, Winter above all rejects metaphysics as a mind game that cannot contribute to philosophy, let alone science. This categorical rejection of metaphysics is expressed many times by Winter, and is summed up in these words: "Let us therefore leave symbolism to the poets, but let us not expect to find in it a philosophical method" ([1], p. 179).

Winter remains constantly suspicious of the unfounded drift of arguments and assertions, the stealthy drift of philosophy towards the arbitrary postulates of metaphysics. The origin of this arbitrariness is

the indeterminacy and indeterminability of metaphysical postulates, which are all as valid as any other, and it is therefore the infinity of possible unifying principles for metaphysics that condemn it by its inability to decide rationally, as seen in the sterile speculations on different infinities ([12]). Winter thus believes that "extreme wisdom for a mathematician would be not to consider such problems, at least in their absolutely general form" ([12], p. 611), but to judge them in the light of their usefulness in the real world of science and its applications, a criterion of truth already raised.

Metaphysics: a fuzzy posture. The second notable argument against metaphysics is the lack of precision in its arguments and reasoning, which is equally condemnable, since vague arguments forbid to prove anything clearly, especially not to discuss anything precisely. Thus Winter rejects Kant's general logic as absolutely disconnected from reality and de facto empty of any content, and his metaphysical discourse on transcendental logic can only discredit from the outset a thought that could otherwise be worth considering ([3]). Generally speaking, metaphysics falls into all the anti-positive pitfalls that emerging logic highlights and helps to avoid: fuzzy or tautological propositions, through its ill-defined concepts, and the burden of arbitrariness that guides the choice of postulates for reflection, making metaphysics the abyss *par excellence* of "lawyer proofs" that Kant himself seeks to avoid. By banishing both mathematics and experience from its reasoning, metaphysics deprives itself of demonstration, "but, without demonstration, there is no valid intellectual theory, and it is this inability to provide proofs that constitutes the vice of all metaphysical philosophy" ([3], p. 604). The father of positivism, Comte himself, sinks into the vagueness of generalities with the ambition for positivism to grasp the reigning order in the fundamental notions of science, considered as above science and resulting in an "arbitrary reversal of method, in the realm [of] generalities [that] have been completely useless to science" ([7], p. 325). Generally speaking, "the regressive method, the metaphysical analysis of elementary notions, is a return to the vague forms of the vulgar discourse of common thought, from which science originated [...] and it is therefore sterile work" [...] ([8], p. 914) to tolerate a metaphysics that represents nothing more than a regression of thought towards the vagueness that modern positive science seems to have definitively conquered.

This ambient vagueness, firmly anchored in philosophical practices, has both a reassuring and a deceptive effect, affecting even the most rigorous scientists. Thus, like many of the great rationalists and scientists such as Kant and Poincaré, the vagueness of metaphysics perniciously intrudes into discourses that nevertheless claim to be of a different rigor: Poincaré's defense of intuition over logic, blandly immune to the attacks on any lack of scientific knowledge, witnesses the difficulty of the struggle. Winter's choice is to rescue philosophy from this obscure and empty metaphysics, and in fact to liberate science from it, because "for [philosophical] reflection [on the sciences] to be effective, it must be oriented in the very direction of science; it must strive to advance positive science with its own means, and not lose its way in vague theories of knowledge, which are generally no more than theories of ignorance ([7], p. 345). This is the case, for example, of "intuition [which is] a certain metaphysical principle whose scientific definition cannot be given, but which can, to a certain extent, be determined in the following way: intuition is transcendent with respect to purely logical forms" ([9], p. 60). Throughout his critical commentaries and historical studies, Winter meticulously strives to detect the metaphysical parts of the arguments. For example, he condemns Émile Picard's surreptitiously metaphysical argument, which rejects without concession the idea of hereditary mechanics, even though, in order to dismiss hereditary explanations a priori, "they would have to contain some kind of crippling logical flaw, from which other methods would be exempt. However, this does not appear clearly" ([14], p. 275), and the conclusion is more the expression of a vague and illusory conviction than of a valid argument.

Uselessness of metaphysics for the philosophy of science. Despite the presence of metaphysics in philosophical reflections on science, Winter does not overly elaborate much on a topic that, dealing with the metaphysical, can only remain sterile. The scientific reality rem ains indeed little affected by these questions, since they will never, as Borel argues with regard to the axiom of choice ([12], p. 615), influence scientific practice and the search for solutions: these are far more important issues than the metaphysical illusion that has haunted philosophers since antiquity. It is through historical and critical study that Winter tackles this question, and notes the uselessness of metaphysics in scientific work.

The history of wave mechanics, traced by Winter in his brief study of Louis de Broglie's *Introduction aux principes de la mécanique ondulatoire* ([20]), is a perfect example legitimizing the philosopher's positions. Indeed, it was by identifying the principles underlying the analogies observed that geometrical mechanics evolved into wave mechanics, as a result of the evolution — this time purely scientific — of the notion of light and the principles of geometrical optics. The analogy between the two theories of mechanics and optics paid off, as the evolution of optics naturally led to a revision of the fundamental positions of mechanics. The new theory has distinguished itself by its efficiency and its agreement with the results of other fields of physics, therefore finding a far more legitimate position than its predecessor, both in terms of its logical status and its usefulness. The failure of metaphysics to deal in the most abstract way with the problems of physics is further reinforced, for "there is [in this new theory] a state of things which numerous experiments and calculations have imposed, and against which general reasoning is powerless" ([20], p. 131), while metaphysical discourse have instead given birth to monstrosities so difficult to destroy, for instance ether, humours, phlogiston, etc. This is a rational and critical philosophy, based on the analysis of the evolution of ideas and the logical genesis of the principles of science and its various branches, exploiting the fruitfulness of the analogy between mechanics and optics — which is a method internal to science — that imposes itself against a vague and abstract metaphysics: analogy is thus legitimized, becoming a valid epistemological tool.

The previous example is already an indication that science now has the tools to think out, clarify and organize its own principles, with clarity and fruit, fulfilling the aim of the Winterian philosophy of science. Metaphysics will naturally continue to wither away little by little with the advance of science, for "the boundaries [of metaphysics and science] are not determined for eternity, those of the former narrow, those of the latter widen with the development of scientific thought" ([3], p. 591). Indeed, the metaphysical philosophers who claimed to be the only legitimate ones to study the principles of science have lost their monopoly to modern logicians and scientists, and "it must be acknowledged today that metaphysical doctrines, elaborated with transcendent ulterior motives, have done little to illuminate and determine the foundations of science" ([3], p.. 589), in contrast to the undeniable success of analysts and geometers, who

"not only extended the proper domain of science, but [...] controlled and fixed its fundamental principles" ([3], p. 589). If, as Winter proposes, the metaphysics of science aims "1) at finding a philosophical foundation for the principles of science, and 2) at unifying human knowledge" ([3], p. 598), history seems to have removed all *raison d'être* for this metaphysics. Logical developments provide a solution to the first problem — never definitive, but sufficient for the criterion, the only legitimate one in science, of utility and experimental verification ([2], p. 616) — and the critical history of science provides a solution to the second. All light can thus be shed by science alone, by means of definitions clarifying vulgar notions, but this demonstration of principles is eminently scientific and avoids all metaphysics ([3], p. 600). The millennia-old and stagnant enterprise of the metaphysicians finally seems to have found its fulfillment ironically in the hands of Boole and Russell, logicians and great opponents of metaphysics, and if "numbers form, as it were, the very armature of intelligence" ([7], p. 345), the goals of metaphysics could be entirely fulfilled by science.

The anti-metaphysical conclusion is reached not through a new metaphysics, but through the simple historical-critical observation that metaphysics has remained unable to solve its problems, and that modern science and logic are fruitful in their own right. This is Winter's constant approach in his research: to legitimize approaches through the concrete utility and historical development of ideas, because " we think with the current forms of thought and the question of whether they have always existed or will always exist is meaningless: we cannot answer the problem of origin" ([2], p. 618). So, like the problem of infinities and the opposition between logicians and intuitionists, we cannot settle the question, we can only show its various sides. He who wishes to formulate absolute conclusions exposes himself to inconsistency in the present, and to being contradicted by facts in the future " ([12], p. 616). This is a philosophy based on historical epistemology, judging by the quality of scientific achievements, according to what a conceptual Darwinism would be.

Winter, holding the torch for an entire positivist generation of philosophers and scientists, seems victorious against metaphysics, both in philosophical reasoning and in scientific thought, not least because metaphysics is out of its object. However, this general

anti-metaphysical fervor conceals a tendency towards the systematic logicization of the world, and now that logic's great adversary seems defeated, a logicist imperialism is taking shape.

§ 4. — Winter against logicism.

The metaphysical arbitrariness of logicism. The dazzling logicization of the sciences at the dawn of the 20th century is presented by its followers as the pinnacle of the rationalization of thought. This revival of logic, with its all-encompassing formalism, is akin to the realization of Leibniz's dream of *calculus ratiocinator*, which reduces all philosophical or mathematical discussion to algorithmic calculation. Logicians have therefore "not only extended the proper domain of science, they have controlled and fixed its fundamental principles. [...] Until then, no philosopher had thought of testing the value of Euclid's postulatum and the principles on which his geometry is based" ([3], p. 589) This new logic is fundamental in that it claims to be the foundation of the other branches of science, becoming the language through which the sciences express themselves, the form of their discourse. It is the precise study of the laws of this discourse that Winter calls *logisitics* ([7], p. 321).

However, despite this powerful and fruitful logical revolution, unlike metaphysics, which theses have little influence on positive science — the foundations of logic being, by extension, the foundations of scientific reasoning — biased logic can only lead to a deviation in science itself. In opposition to the positivist and logicist tendencies of his time, Winter does not fail to identify shortcomings he already criticizes in metaphysics. It is therefore appropriate to raise very precisely the arguments behind Winter's metaphysics accusations against logicism.

Winter's first criticism of logicism, analogous to his criticism of metaphysics, is its arbitrariness. Indeed, the logical systems that claim to govern thought exist in an infinite variety: almost any variation of axioms remain an admissible logic system. This can be seen in the multiplication of consistent logical systems, such as the different geometries, which are nonetheless exclusive. This multitude of possibilities requires a choice that can only be based on metaphysical arguments, practical observations or cultural habits. For over two millennia, Euclidean geometry thus arbitrarily closed the door

to other geometries. These arbitrary choices are all the more serious in that "given the tacit assumption that logical-mathematical laws have an absolute value, we apply to them the idealist principle [that] the laws of the mind are the laws of things", and the arbitrary formalization effected by logic is therefore impoverishing ([2]) for science, and binds it to principles from which it will find it difficult to free itself. These axioms, atoms of logical systems, are to be discussed in a particularly enlightened way: "from the moment that the progress of mathematical methods consists in giving logical rigor and precision an ever-increasing share, we must necessarily be led to scrutinize the very foundations of the reasoning of geometers, to analyze the fundamental types of demonstrations, and to fix the indefinable elements on which these demonstrations rest." ([5], p. 189).

Even more than the arbitrary choice of the axiomatic system, this formalization is abusive, and assumes without justification the possibility of translating an obviously fuzzy thought into a precise and perfectly delimited calculation. Thus, the general craze masks the impact on physical and mathematical models of our choice for classical logic, which has been so reluctant to introduce approximations in analysis or statistics in physics: "isn't there an obvious contradiction in wanting to give an absolute logical foundation to what is only psychological and human?" ([5], p. 198), and isn't it condemning science to do so? This confirms Winter's anti-logicist stance. He does, however, recognize the merits of logic, and hardly condemns science to the status of mere speculation. If reasoning is to be governed by rules, these rules must be clarified, evolve and be perpetually questioned, in the light of their usefulness, which alone guarantees the value of the choices made.

Winter does not fail to raise these limitations in several of his historical studies. In particular, he notes that "Hilbert's axioms are purely formal in character and are given as a series of arbitrary decrees [and] this is the flaw in this doctrine, whose logical importance nevertheless remains great. The author develops a hypothetical-deductive system of axioms for the science of the moment, and it is not obvious that another system of axioms cannot be substituted for this one to some extent" ([19], p. 230). Hilbert himself would fall into the same trap by not doubting the axioms of classical logic, which are just as arbitrary a priori, although he does mention the legitimacy of the principles of equality and syllogism in [2] (p. 607). The infinity of possible postulates that so

often characterizes the terms in which a metaphysical problem is posed is found in the fields of knowledge that logicism claims to subject to its uniformity. This limitation is particularly evident in the interpretation of legal texts, for which logic will never be able to determine a criterion for choosing among [the] postulates" ([4], p. 618). In the same vein, the great debates between logicians and philosophers are also all metaphysical in nature, unduly monopolizing attention and effort. So the relationship between logic and the reality of thought, such as the debate between purely logical reasoning and the existence of an intuition irreducible to logic ([5]), does not appear to be proven, and is at the very least not reducible to one another. Thus, he concludes that logistics, as a universal explanation, must be condemned in the same way as all metaphysics, because exercised outside its field of application, it remains a pure mind game with no scientific utility" ([5], p. 215). This rejection of logic's claim to universality heralds the failures of a future *Tractacus* to logicize the world.

The utility criterion for judging logicism. The logicist tendency to believe that, "by following the laws of logic, all truths can be found" ([9], p. 924) is refuted by Winter, since no logician has ever succeeded in reconstructing any part of science autonomously. The intuitionist, hardly in a better position, is confined to advancing the truism that "the inventor is not a calculating machine" and defending a certain metaphysical transcendence of intuition. The debate between these two positions, which appears to be insoluble, has no substance, and the two positions "are in fact opposing metaphysical theses whose very nature is that they can never be demonstrated scientifically and peremptorily" ([9], p. 925). The only reasonable, and never definitive, positions we can hope to advance on this issue are those based on *usefulness* alone.

This criterion of usefulness is enough to disqualify logicism, because the advent of logic and its systematic application has not solved the majority of problems facing the sciences. Indeed, if "formal logic cannot account for" ([5], p.. 207) certain mathematical phenomena, such as the sudden appearance of certain properties like the non-solubility of polynomial equations from degree 5 upwards, this is proof that logic cannot make up for the completeness of science, because "in the face of real mathematical difficulties [...] formal logic [is] of no use" ([5], p. 209). Or, on the other hand, it cannot independently resolve problems arising from science: The

question of how to represent an integral raises a series of mathematical problems. For every function, there is an infinite number of expressions that can be used to represent it. The expression we choose should highlight the function's characteristic properties as accurately as possible, and in particular its singularities. However, among this infinite number of expressions to choose from, logistics offers no positive criterion for deciding which is the best. As we have already seen in number theory, the mathematician is faced with real mathematical difficulties, against which formal logic is of no use." ([5], p. 209)

At the end of the day, "to try, as has sometimes been attempted, to establish that [logic] is, absolutely speaking, superior [to the rest of science], seems to us a meaningless undertaking, because we are in the presence of two equally fundamental ways of seeing" ([13], p. 467), each of which complements the other. The asserted criterion of utility ([5], p. 202) also works against logic *per se*, because logic has never succeeded in constructing anything on its own ([9]), and the great scientific revolutions have never resulted from logic alone ([10]). Such revolutions are first and foremost the work of philosophers, not logicians. Logic must therefore remain — as Russell concedes, even though he himself was one of the instigators of the logicist ideal — a branch of mathematics, ignored in fact by most mathematicians, and not the indispensable instrument of all mathematical research ([5], p. 189).

Logic as a control tool. There's no denying the fruitfulness of logic's ideas and what this degree of formalization and abstraction has brought ([13], p. 473 ff., but also [5], [10] and [17]). This canon imposed on reasoning is, when used in its role as keeper of rigor and clarity, the mean of guarding against inaccuracies and general a priori that have no foundation. Indeed, logic, "the axiomatic analysis referred to in this note, is an extension of Descartes' method [...] It essentially consists in substituting intuitive and experimental notions, often confused, with clear and distinct ideas" ([16], p. 28), reflecting both the Cartesian ideal of clarity as one of the primary criteria of scientificity, and the Winterian philosophical objective of clarifying and ordering principles. The value of asserting the positive virtues of logic goes beyond mathematics, and Winter shows its applicability on a simple but generic example of legal controversy, where the ambiguity of laws confirms that

"only the precise use of algebraic formulas can avoid similar confusions" ([4], p. 618) between logical principles. The impossibility of applying the methods of the exact sciences to law is the simple consequence of the absence of rigorous reasoning and a precise critical method. Thus, "confusion of opposites and contradictions, and ignorance of Morgan's laws" ([4], p. 623), led the highest legal authorities and the most experienced legal doctors end up deciding in favor of the least consistent solutions, as Winter's logical study of the legal controversy in [4] shows. The evolution of analysis has thus "[substituted for vague intuition] exact analytic definitions. We can therefore say, in keeping with historical truth, that the analysis of the 18th century was much less rigorous than that of the 19th century, and unless we maintain that the work of the 19th century marks a step backwards on that of the 18th century, we must recognize that the overall evolution of mathematical science indicates progress in the direction of greater logical precision" ([5], p. 188-189). The advent of modern logic therefore makes it possible to have a perfect *organon* governing proofs.

This new status for logic, central to modern science, raises questions of its own, and Winter recognizes that logicists have a necessary clarifying role to play here. "To teach the first elements of calculus, it was necessary to use sentences; indeed, it would be impossible to explain the beginnings of mathematics without a prior grammatical discourse. [...] It is these [grammatical] questions which must be answered, and which must be treated with the same precision as mathematics itself, if the starting point of arithmetic, analysis and the other branches of mathematics is not to be drowned in a fog. [...] As we cannot imagine what the mind of a man would be like, from which all grammatical-logical notions would be banished, the very principle of all positive philosophy obliges us to study these notions as given facts, to establish the laws of combinations that are peculiar to them, as well as the relationships that unite them to mathematical science proper. This is the positive field of application of Logistics, a field which constitutes the logical introduction to the theory of numbers and the theory of functions." ([5], p. 195-196). In other words, "the analysis of mathematical reasoning, i.e. the determination of the grammatical-logical types it contains, is the proper domain of logistics" (*ibid.*, p. 197). This is a central task for the mathematician and philosopher alike, but one that will never be completed: "the system of our logical ideas is a closed system, and [...] the universe of abstract beings

is an open system. But, by this very fact, the study of the universe of beings will never be completed, and it will force us to modify the closed system of logical constants." ([5], p. 210). Logic, as a study of the grammatical discourse of science, becomes central to scientific reasoning, and is thus a subject of study in its own right.

Logic naturally forms the last stage of the "process of thought essential to the exposition of mathematics" ([7], p. 322). This final stage is extensively studied by Winter, in the case of Frege's logical foundations of number theory or Hilbert's axiomatization of physics. Logic thus enables a certain unification of mathematics ([5]), in its language component, and it is an abstraction which, by distancing itself from its sensible object, frees us from vulgar a priori and avoids the illusions of the real and the familiar, of examples and intuitions ([16]). Logic thus finds a place within science, as a particular branch with circumscribed aims rather than as a specious one that would be above it.

The importance of the link to reality. To adapt to the problems discovered by time and practice, logic needs to be enriched with new definitions. These new objects of thought are arbitrary, but legitimized by their usefulness and natural necessity ([5] and [15]). Indeed, if science were to seek only generality and abstraction for its own sake, it would lose its ties with the particular things it sets out to explain, a situation that is obviously unbearable, for by developing purely abstract theories for their own sake, "we would run the risk of straying into the realm of formal logic and scholasticism" ([13], p. 509), in effect abandoning the primary objective of science: understanding the real world. Thus Cantor's thoughts on the powers of infinities seem, in Winter's eyes, to belong more to the realm of contentlessness than to the heart of science, and "if transfinites cannot be rejected by virtue of an argument from formal logic [...] only the use that can be made of them in particular problems will decide their scientific value" ([12], p. 615). Similarly, "the logistical definition of the irrational and the considerations that logisticians have made about it, whether or not they have been put into formulas, shed no light on the determination of the distinctive characteristics of commensurable numbers and incommensurables, of algebraic incommensurables of different degrees, and finally of transcendental numbers. The results obtained in these areas were achieved by directly tackling the difficulties using classical mathematical methods." ([5], p. 202). Another example is provided by

his study of the meaning of the deterministic principle in physics, "which takes place by examining how this principle comes into play in the solution of problems" ([13], p. 74), thus becoming deeply rooted in the very practice of science.

According to Poincaré, as for many other mathematicians and physicists, experiments thus become "not a control procedure, but a method of demonstration" ([3], p. 615, in stark contrast to the positions of both the *Revue* and the logicists) in its own right. The logicist tendency reduces the philosophical or mathematical problem to a single aspect, whereas "philosophy must also determine the constitutive and specific differences in the principles of the methods" ([2], p. 610), which alone will make it possible to embrace the entire problem. Attachments to reality therefore seem paramount, and "we thus recognize that scientific judgment needs a certain particular content; but in trying to establish that this content is external to its method, we preserve that same method's universal scope" ([1], p. 172). Science is founded on coherence between its parts and, like the mason who must build his edifice from bottom to top," the logic of science can only pride itself on being a demonstrative and objective doctrine the day it becomes an integral part of science" ([3], p. 603), and if it can and must distance itself from its object and abstract itself in order to retain a sufficiently broad scope to serve as a general scientific method, it cannot cut itself off from it entirely, for it is its *raison d'être*. Experience, and in particular its basis in calculation and experimental logic, thus appears as a central method that should not be rejected without foundation.

If logic becomes the organon of scientific discourse, its critical and reflexive study remains subject to confrontation with the objects and problems internal to science. So, "we see the possibility of an axiomatic analysis of physics. We say analysis, not deduction of principles. It would be an analysis *a posteriori* of scientific notions from a given science in fact, and not a construction *a priori* of science. In the case of physics, the compatibility of axioms results above all from experimental verification." ([13], p. 101). "The system of physics axioms must be conceived as an experimental-deductive system. The expression "hypothetico-deductive system" coined by logicians does not seem to us to express clearly enough that the sole purpose of postulates and hypotheses is to introduce experimental data into calculations. From the point of view of formal logic, we can forget for a moment, in the manner of Hilbert, the intimate links

that unite fundamental principles with experience; we can't do this from either a physical or a philosophical point of view and, in the end, we have to conceive of the axiomatics of physics that we have just sketched as the logical analysis of experience." ([13], p. 102). All this philosophy looking through experience is summed up by Winter in these words: "the theory was born on a land of experiences, and must remain there to develop further" ([18], p. 323).

Jacques Hadamard is presented by Winter as another perfect example of a theoretical and rigorous mathematician who does not neglect this essential aspect that gives meaning to the mathematical theories and ideas he expounds in his courses at the École Polytechnique, "in a constant concern to link analysis to its true source: Physics" ([21], p. 608). Only this marriage between the abstraction of methods and the reality of the object can guarantee the fruitfulness of science, a lesson from history defended by Winter and well-illustrated by Hadamard. As Winter points out, while mathematical analysis has contributed much to the development and formalization of physical models, physics has been no less a driving force behind the development of analysis, and to forget this perpetual mutual enrichment would be to deny the very origin of ideas. It is these numerous applications [that] bear witness to the fruitfulness of new ideas; they are, in a way, the guarantee of their objectivity, and "criticism [must keep] the content in its experimental form to prevent metaphysical principles, mere conditions of thought, from being transformed into dialectical ideas" ([2], p. 619).

Science is thus always nested in its object, at the confluence of many other fields ([24]), and to forget this would be to deny a part of science, perhaps the only part that links it to reality and differentiates it from purely speculative philosophy, from metaphysics. So analysis, as Winter develops in his historical-critical study of the evolution of function theory, must not forget its object — physics — and it is by remaining linked to its object that it will retain its fruitfulness and continue to evolve ([5] but also [17].). Winter thus circumscribes "logistics [which] must have boundaries in the sense of metaphysical abstraction (an upper limit, if you like), which means that it must, through an explicitly formulated postulate, cut short any digression into the sterile realm of scholasticism. It must also have boundaries in the sense of its applications (lower limits) [...] to avoid any duplication with already existing methods." ([5], p. 190).

Although logic, in its positive and non-metaphysical role, appears to be the best canon for presenting science and a rich guardian of the rigor of reasoning, it is not enough to solve scientific problems, and its use must be that of a conscientious, powerful but epistemologically limited tool. In contrast to the logicist tendency of his time, Winter emphasizes the philosopher's role in guaranteeing the balance between logical freedom and the reality of science, synthesizing the forms of laws and the reality of objects.

§ 5. — Conclusion: A historical-critical epistemology rooted in the sciences.

An original mathematical philosophy. Neither philosopher nor mathematician, Maximilien Winter eschewed the major trends of his time in both metaphysics and logicism, in favor of a rational, critical philosophy rooted in the sciences and their history. In this respect, he set himself apart from his comrades from the *Revue*, whose aim was to rehabilitate metaphysics and morale, and from the vast logicist school developing in Europe. Discreetly and unpretentiously, he opts for a relatively new philosophical ideal, denying philosophy its primacy over thought and putting it back on a par with mathematics, science and facts, without reducing it to them — avoiding a positivist stance — so that each can shed light on the other. Philosophy of science thus becomes a scientific philosophy, where problems are no longer purely abstract, but where thinking revolves around science. This approach makes it possible to identify fundamental and fruitful principles, never definitive, but whose value is guaranteed both internally, through their usefulness within science and their relevance to its objects; and externally, through the historical-critical confirmation of their a posteriori fruitfulness. The coexistence of science and philosophy is thus made possible by a dialectical organization between the two, a position giving superiority neither of philosophy on science (as would be the case with metaphysics), nor of science on philosophy (as would be the case with logicism or positivism), but a mutually profitable exchange — science becoming aware of its methods and principles, philosophy being enriched by scientific technique and issues.

A timeless choice. With his pious wish to limit pretensions and balance the roles of both philosophy and science, Winter remained philosophically isolated throughout his life, his studies and ideas finding few echoes and quickly forgotten after his death. His reflections were swept up in a torrent of enthusiasm about the foundations and principles of science, founding the logicist wave that would sustain both the Vienna Circle in the philosophical world and the Bourbaki group in mathematical spheres. The man who advocated impartial judgment of science in the hope that it would continue to progress, in the light of rational philosophy and critical history, found himself overshadowed by the positions he opposed. More than half a century later, efforts and hopes to base science and the world on a definitive, closed system of logic collapsed. Gödel's results intersected with Winter's departure, and although he found strong scientific support for his rejection of logicism, logic continued its quest to standardize mathematics, and the philosophy of science essentially continued to detach itself from science, borrowing only its most arbitrary part: its new logical formalism.

The judgment of history. A century of evolution has seen the gradual collapse of logicist enterprises. If logic has provided a powerful and fruitful tool for science and philosophy, as Winter defended, the desire to condense facts and sciences has impoverished theories and ideas, standardizing them in an artificial and arbitrary way, which the philosopher already decried as the usual metaphysical reversal of reasoning: the image we have of the world has become a postulate from which we seek to explain a world that is no longer the same.

The incredible growth of applied mathematics has brought as much to the other sciences as these applications have brought to mathematics, with Kantorovich's economics fruitfully resurrecting Monge's twice-century-old transport problem; Yves Meyer's image processing bringing a revolution in the theory of function representations from Fourier series developments thanks to wavelets ; biological and financial modelling, creating the conditions for the development of random processes, which had not seen major advances since the amusements of the Chevalier de Méré; computer security, giving that simple game of the mind that was arithmetic a new lease of life with ubiquitous applications; and no one will forget the contributions of the development of aerodynamics and design to geometry. Philosophy also contributes to this enrichment

of mathematics, Carlo Rovelli's philosophical conceptions of time and space having been the very essence of Alain Connes' developments in non-commutative geometry. Never complete, the task of science thus always advances in the right direction, "slowly but surely moving towards an ever more perfect order" ([3], p. 606), epitomizing Winter's optimism. The evolution of metaphysical systems gives way to the progress of science, which is based on, and illuminated by, logic and critical philosophy.

Maximilien Winter, that illustrious unknown who believed in a living mathematics and philosophy and who defended as the only legitimate judgment that based on critical history, that jurist ignored by those he hoped to enlighten and who preferred to embrace the logicism and metaphysics he rejected, will have departed in the shadow in which he lived. Today, his philosophy has shown itself capable of guiding and enriching the sciences, his own historical-critical method having been taken up by some of the greatest mathematicians trained within the Bourbachian opposition.

Another jurist ahead of his time had once illuminated mathematics and paved the way for future generations, and it may be a century later that we realize what Maximilien Winter contributed to the French mathematical epistemology.

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