Introduction

This special issue[1] in two volumes is devoted to a certain French tradition in the philosophy of mathematics, a tradition characterized by the tutelary presence of two major figures, Jean Cavaillès (1903-1944) and Albert Lautman (1908-1944), both of whom were shot by the Nazi occupiers for their involvement in the French resistance, and both of whom produced original, albeit unfinished, works in the course of their short lives. The close dialogue between these two authors throughout their short lives delineated a theoretical space where points of agreement also gave way to fundamental disagreement. Among the main points of agreement was the idea of the need to focus on an effective understanding of the mathematical practice of their time[2] (experience, as they both call it), in particular by entering into a concrete dialogue with leading mathematicians. Secondly, the thesis that the true nature of mathematics is to be a "becoming". This implies that there can be no real understanding of mathematics if we neglect its history. Finally, the idea that the central philosophical question that guides philosophers in this task should be the problem of the meaning of this becoming, of what it tells us about reality and about our reason. So, it is not simply a question of a philosophy of mathematics, in the same way as there is a philosophy of physics or biology, each of which would be regional epistemologies, but rather of a Mathematical Philosophy (*Philosophie mathématique*), that is to say a reflection on the lessons that philosophy of knowledge must draw from mathematical experience. The disagreements, on which we will focus below, concern the philosophical meaning that should be attributed to this development : philosophical meaning that touches directly on the problem of

[1]The authors would like to warmly thank Jean-Pierre Marquis for his suggestions, which helped to clarify this introduction.

[2]The characterization of the effectiveness of this knowledge that Cavaillès gave in his work constituted a kind of standard of requirement for the French school in the philosophy of mathematics, and more generally in the philosophy of science; the philosopher must not be satisfied with abbreviated or simplified accounts of mathematical work: he must be capable of "catching the gesture" (Cavaillès 1981, p. 178), that is to say of following the genesis, the explanation and the justification that mathematics gives in its own singular and concrete becoming.

the relationship between mathematics, history and philosophy. This theoretical area of debate, what inspired it and what it inspired in the next generation, is the subject of this double issue.

This first issue opens with a full English translation of the meeting of the *Société française de philosophie* on 4 February 1939, devoted to the presentation of the theses of Cavaillès(3) and Lautman(4) respectively, followed by a debate attended by several mathematicians, including Elie Cartan, Charles Ehresmann, Paul Lévy, Maurice Fréchet and others. Then, in the two volumes, we present a series of articles which, without any claim to exhaustiveness, discuss certain aspects of the philosophical work of Cavaillès and Lautman, as well as the conceptions of several authors around them. Léon Brunschvicg (1869-1944), Cavaillès and Lautman's thesis supervisor. Jules Vuillemin (1920-2001), Gilles Gaston Granger (1920-2016) and Jean Toussaint Desanti (1914-2002), who attended classes with Cavaillès at the École normale supérieure and listened to Lautman there. As a counterpoint, we add an article on Maximilien Winter (1871-1935), a contributor like Brunschvicg to the *Revue de métaphysique et de morale*, but whose interest in the history and practice of mathematics is tinged with a greater presence of positivist themes.

The term "philosophy of the concept" has long been used in France to designate, following Cavaillès, part of the tradition of French philosophy of mathematics which proposes a third way between analytic philosophy and Husserlian phenomenology. The origin of this term can be found in the very last lines of the posthumous *Sur la logique et la théorie de la science* (edited by G. Canguilhem and Ch. Ehresmann in 1946), in which Cavaillès states that "It is not a philosophy of consciousness but a philosophy of the concept that can give a doctrine of science. The generating necessity is not that of an activity but of a dialectic". In 1946, Canguilhem emphasized the enigmatic nature of these last words of Cavaillès's intellectual testament, but in 1976, at a time when structuralist currents were in full swing in France, he added: "We can understand today that the enigma was tantamount to a foreshadowing; Cavaillès assigned,

(3) Cavaillès defended his theses at the Sorbonne (Méthode axiomatique et formalisme (main thesis) and Remarques sur la formation de la théorie abstraite des ensembles) in Jannuary 1938, under the supervision of Léon Brunschvicg.

(4) Lautman defended his theses in December 1937 at the Sorbonne, also under the direction of Léon Brunschvicg. These were Essai sur les notions de structure et d'existence en mathématiques (main thesis) and Essai sur l'unité des sciences mathématiques dans leur développement (essay on the unity of the mathematical sciences in their development).

twenty years in advance, the task that philosophy is in the process of recognizing for itself today: to replace the primacy of lived or reflected consciousness with the primacy of the concept, the system or the structure" [Canguilhem 1976, p. 32].(5)

However, the interpretation of this passage is far from unanimous and many works devoted to Cavaillès have discussed, and are still discussing, these last lines. In fact, this interpretation calls for a diagnosis of the posterity of Cavaillès's work and therefore (a rather thorny question) of the perimeter of those who can be considered to have prolonged his unfinished work.(6) It is significant that recently some (see, for example, Alain Badiou, *L'aventure de la philosophie française*) have gone so far as to speak of a Brunschvicg school (which would include, according to Badiou, Lévi-Strauss and Althusser) that would confront the Bergsonian school in such a way that their opposition would have marked French philosophy as a whole from the beginning of the twentieth century, precisely as an opposition between the philosophy of consciousness and the philosophy of the concept.(7)

(5)This same term was later used by Michel Foucault in his introduction to the American edition of Canguilhem's book *Le normal et le pathologique*, to contrast the French philosophers of science (the philosophers of the concept), almost all of whom were involved in the Resistance, with the philosophers of consciousness who, like Sartre and his existentialist friends, were more concerned with their own work during the Occupation.

(6)See the analyses by Houria Benis Sinaceur (in: *Jean Cavaillès : philosophie mathématique*, Presses universitaires de France, 1994 and *Cavaillès*, Les belles lettres, 2013) and Elisabeth Schwartz (in particular "Jean Cavaillès et la philosophie du concept", *Philosophia Scientiæ*, no. 3, 1998, pp. 79-97 and the two issues she edited, "Jean Cavaillès 1", *Revue de métaphysique et de morale*, no. 105, 2020, and "Jean Cavaillès 2", *Revue de métaphysique et de morale*, no. 106, 2020).

(7)"In 1911, Bergson gave two very famous lectures in Oxford, later published in the collection La Pensée et le mouvant. In 1912, Brunschvicg's book Les Étapes de la philosophie mathématique was published. These two interventions (just before the 14-18 war, which is not insignificant) set thought in directions that were, on the surface at least, completely opposite. Bergson proposed a philosophy of vital interiority, subsumed by the ontological thesis of the identity of being and change based on modern biology. This orientation was to be followed throughout the century, up to and including Deleuze. Brunschvicg proposed a philosophy of the concept, or more precisely of conceptual intuition (a fruitful oxymoron since Descartes), based on mathematics, and describing the historical constitution of symbolisms in which fundamental conceptual intuitions are in some way collected. This orientation too, which ties subjective intuition to symbolic formalisms, continued throughout the twentieth century." A. Badiou, *L'aventure de la philosophie française*, La Fabrique editions p. 9. In the wake of Badiou, see also Tryggvi Örn Úlfsson, *D'une épistémologie mathématique vers une ontologie phénoménologique : Séquences de la philosophie du concept du 20e siècle* ; Thèse de doctorat de l'Université

We will not enter into this debate, wishing to remain focused in this presentation on this current of French epistemology which has made specific reflection on mathematics the central point of its own philosophical approach. We do not claim to trace the contours of a school, but, as we have said, to indicate the perimeter of a debate, even a contradictory one, which seems to us to be fruitful, which was structured by Cavaillès and Lautman and which has been extended, actually or virtually, by those who have been inspired by their philosophy (Granger, Vuillemin and Desanti in particular).(8)

Hence the dual purpose of these two volumes.

The first aim, is to raise awareness of this movement which, despite the recent revival of interest in French epistemology and philosophy, is still little known outside a restricted field of specialists. It is true that a series of works in English have recently appeared, filling a glaring gap(9), but it has to be said that the American analytic tradition continues largely to ignore the names of Cavaillès and Lautman and the importance that their teaching and of their work had on the whole generation of French philosophers of mathematics in the second half of the twentieth century and until now. In this respect, the *Stanford Encyclopedia of philosophy* is symptomatic. There are entries on Bergson, Poincaré, Duhem, Althusser and Foucault, but not on Cavaillès or Lautman, and no mention of Vuillemin, or Desanti,(10) nor any specific entry

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Paris VIII, Mars 2024. Úlfsson does not mention Granger and Vuillemin in this tradition, but does include Bachelard and Badiou.

(8) In fact, all the implications of their debate on the relationship between history, philosophy and mathematics (for example, the question of objectivity, necessity, applicability and the unity of mathematics in its historical development) were explored by these students...

(9) We are thinking here of A. Brenner, J. Gayon *French studies in philosophy of science*, (2009) Contemporary Research in France, Springer, Vienna/New York, A. Brenner, Epistemology Historicized: The French New Directions in the Philosophy of Science, (2014). In: Galavotti, M., Dieks, D., Gonzalez, W., Hartmann, S., Uebel, T., Weber, M. (eds) The Philosophy of Science in a European Perspective, vol 5. Springer, as well as monographs such as Pietro Terzi *Rediscovering Léon Brunschvicg' critical idealism* (2022), Tiles, Mary, *Bachelard: Science and Objectivity*, Cambridge University Press, (1984) as well as reprints and translations of Lautman's texts (*Mathematics, Ideas and the Physical Real*, 2011 - Bloomsbury Publishing, the new English translation of Cavaillès's *Sur La Logique et la théorie de la science* with preface by Knox Peden, in 2021. (Cavailles, Jean. On logic and the theory of science, (2021). New York, NY: Sequence Press. Edited by Knox Peden & Robin Mackay)

(10) There is, however, an occasional mention of Granger in the SEP entry on *Style*.

on French historical epistemology (although there are some on China, India and Latin America). This is despite the fact that Paul Bernays published a review of Lautman's thesis *Essai sur Les Notions de Structure et d'Existence en Mathématique* in the *Journal of Symbolic Logic* in 1940[(11)] and that between 1938 and 1940 there were seven reviews of Cavaillès's work in the same journal by A. Church, E. Beth, L. Henkin, E. Nagel and M. Black.

Perhaps a first reason for such a blind spot in the American tradition of philosophy of mathematics lies in the disconnection between it and French epistemology between the 50s and the 70s. At that time, the fundamental tool in the American analytic tradition was formal logic, a tool that was fairly secondary in France during the same period in the field of epistemology. It is a fact that after the Second World War, the focus in America shifted to Gödel's incompleteness theorems, the development of Tarski and Robinson's model theory and its impact on philosophy (see for example Putnam), the development of intuitionistic logic and multivalent logics (very popular in the 1950s and 1960s), the rediscovering of Gentzen's work on natural deduction (via Beth's and Hintikka's tables in the 50s and 60s), the semantics of modal logics, Cohen's forcing in set theory, the introduction of non-standard analysis by Robinson and, finally, higher-order logics. The works of Stephen Kleene, Alonzo Church and Georg Kreisel, with all their profound philosophical implications, have been widely discussed and recognized. In short, analytical philosophy required formal logic, which was quite absent in France at the time, nevertheless with some remarkable exceptions, related to Cavaillès and Lautman's legacy.[(12)] It's a fact that since the 1990s, formal logic has had less impact on philosophy in general, and philosophy of mathematics is opening to other methodological tools.

A second reason for this blind spot in American philosophy of mathematics of analytic orientation lies in the difficulty for it of perceiving the specificity and originality of the French tradition in philosophy of mathematics. The progress made in the historical reconstruction of the debate on the philosophy of science in

[(11)] *The Journal of Symbolic Logic*, (1940), Vol. 5, No. 1, pp. 20-22. Paul Bernays also mentioned Lautman's work to Kurt Gödel in a letter dated 17 September 1965 (S. Feferman et alii *Kurt Gödel Collected Works vol. IV*, p. 242-243.

[(12)] The works of Vuillemin, Granger and their scholars as well as of some of the members of the *Institut d'histoire et de philosophie des sciences et des techniques* in Paris (most of them scholars of Roger Martin) bear witness on that.

the twentieth century, in this great, lively and plural cultural area that was Europe before the Second World War, is certainly undeniable. However, it should be kept in mind that what characterizes this debate is not so much the plurality of languages in which it is expressed, but rather the subtle interplay of borrowings and refusals to borrow (*emprunts et refus d'emprunts*),[13] constitutive of each of the currents that have been expressed. This interplay of borrowings cannot be reduced to national characteristics, and even less to uniform philosophical choices, but is the result of a conscious and argued search for a solution to the problems posed by science. The tradition we are interested in has distanced itself in many ways from its contemporaries. Distancing itself from Poincaré[14] , from the ideas of Abel Rey, Meyerson and the philosophers who founded the Institut d'Histoire des Sciences et des Techniques in Paris, from Bergson, and also from the Husserlian phenomenology, German neo-Kantianism and the Vienna Circle, in particular Carnap, Schlick and Reichenbach. Moreover, this interplay of borrowings and refusals to borrow is nourished by dialogue with the history of philosophy, and in particular the history of French philosophy, characterized by the Cartesian heritage, Comtian positivism, the philosophy of Kant which had penetrated France after post-Kantianism, and contacts with American pragmatism (James and Dewey in particular). We are a long way from having carried out a dispassionate historical analysis of what gave rise to the debate on the philosophy of science and, in particular, contemporary mathematics. Knowledge of this underestimated tradition seems to us to be an important element in developing this history.

The second aim of this double issue is theoretical and methodological, and concerns the fact that knowledge of this tradition has, in our view, an intrinsic value for the current debate in the philosophy of mathematics, marked by the advent of the philosophy of

(13)For the notion of "refusal to borrow" as a factor in the formation of identities, see Marcel Mauss (1920), "*La nation*" : « les sociétés sont en quelque sorte plongées dans un bain de civilisation ; elles vivent d'emprunts ; elles se définissent plutôt par le refus d'emprunt, que par la possibilité d'emprunts. » Extract from Année sociologique, Troisième série, 1953-1954, p37. Text reproduced in Marcel Mauss, *Œuvres. 3. Cohésion sociale et division de la sociologie* (pp. 573-625). Paris: Les Éditions de Minuit, 1969.

(14)Of course, Poincaré's work remains at the heart of the mathematical and epistemological thinking of the Cavaillès-Lautman tradition. However, the absence of any reference to intuition, to a doctrine of faculties or to non-historical a priori forms marks a clear philosophical distance.

mathematical practice movement. It is undeniable that this current is part of a critical study of the "orthodox" epistemological approach typical of the analytic tradition, in which logic, the notion of first-order formal theory, the foundationalist point of view and ontological questions play a prominent role. This new strand of contemporary analytic philosophy thus claims a wider field of reflection than number theory and/or set theory, an attention to the "non-logical" methods intrinsic to actual mathematical practice and to the processes of explanation and justification present in various mathematical fields throughout history, and the need to move away significantly from properly ontological issues and the "quarrels" linked to logicist, intuitionist and formalist positions.

When we speak of an "orthodox" tradition in the philosophy of mathematics of analytic obedience, we mean a tradition that has essentially developed from Quine's philosophy, from his own way of inventing the analytic tradition with clearly identified founding ancestors (essentially Frege, Russell and Carnap), and from the "system"[(15)] that Quine constructed to "renew" Carnapian logical empiricism in a "non-dogmatic" sense. This system clearly includes two principles. The first is the reduction, in the analysis of scientific discourse, of all ontological considerations to the objects of formalized theories. This is the famous principle according to which "to be is to be the value of a variable", which eliminates any entity of an intensional or intentional nature from the universe of discourse of philosophy of mathematics, and has as its consequence extensionalism, i.e., the elimination (in the analysis of proofs based on closed and given a priori sets of axioms and rules) of any consideration other than the establishment of truth. Secondly, the principle of indispensability, which states that the objects that we must recognize as belonging to our ontology of reality are the objects that are indispensable to our best scientific theories, the objects belonging to the domains on which their corresponding formalized theories quantify. To these two principles Quine first adds the doctrine of holism (the idea that the network of our knowledge faces experience as a whole), which deprives the assertion of the reality of the objects on which our theories quantify of any metaphysical value. Holism also implies the idea that radical epistemological ruptures or changes, since they do not affect the

[(15)] We are talking here about a philosophical system in the classical sense of the term: a set of principles and methods from which coherent doctrines are derived.

empirical base which only increases, are merely ontological revolutions and therefore a kind of palace revolution. Knowledge as a whole develops continuously and progressively.(16) Secondly, its naturalism, which has two distinct components: the rejection of all metaphysics ("first philosophy") and the denial of any radical difference between common sense knowledge and science on the one hand, and between philosophy and science on the other.(17) These two aspects were transformed in the Quinian vulgate into the doctrine according to which naturalism asserts that philosophy must be constructed in dialogue with science, which implies that not being a naturalist means ignoring or despising science.(18) This transformation of the principle of Quinian naturalism is obviously false, as shown by twenty-five centuries of philosophy and by the doctrines of many philosophers who have nevertheless attracted the interest of analytic philosophy. Among them, Wittgenstein, for example, who describes what an anti-naturalist position is in the simple aphorism 4.111 of the *Tractatus*:

"Philosophy is not one of the natural sciences. (The word "philosophy" must mean something whose place is above or below the natural sciences, not beside them)."

This expresses the dual place that philosophy has in the *Tractatus*: philosophical work must unfold both as a critique of science (in the broad sense of the term) and therefore below it, and as a radical interpretation (or vision) of the meaning of this human practice, above it.

(16)It is remarkable that the realism/anti-realism debate should have been built around the so-called Quinian Platonism, because Quine, as he himself says, attributes only a theoretical and not a metaphysical value to his criterion of existence: "Reference and ontology thus recede to the status of mere auxiliaries. True sentences, observational and theoretical, are the alpha and the omega of scientific enterprise. They are related by structure, and objects figure as mere nodes of the structure. What particular objects there may be is indifferent to the truth of observation sentences, indifferent to the support they lend to theoretical sentences, indifferent to the success of the theory in its predictions" W. V. Quine, Porsuite of truth, Harvard University Press 1990, p. 31.

(17)"Philosophically I am bound to Dewey by the naturalism that dominated his last three decades. With Dewey I hold that knowledge, mind, and meaning are part of the same world that they have to do with, and that they are to be studied in the same empirical spirit that animates natural science. There is no place for a prior philosophy." W.O. Quine, Ontological relativity, The journal of philosophy, vol LXV, no 7, April 1968, 185.

(18)See David Papinau *Naturalism*'s SEP entry on this subject.

Of course, this orthodoxy began to be challenged in the second half of the 1960s, with the work of Benacerraf and Putnam, but it is symptomatic that each of these challenges, and the currents that developed in their wake, concerned specific aspects of the Quinian system, without ever raising the more general question of a reconsideration of its thinking as a whole and an overall critical reflection on its project.

First of all, structuralism, which developed within the analytic tradition from the famous "What numbers could not be", calls into question all the constraints that Quine had placed on the objects of analysis in the philosophy of mathematics with his foundationalist reductionism to the objects of formalized theories (Quine's first principle mentioned above). However, structuralism (whether eliminative or non-eliminative) does not seem to break with naturalism at all, since the properly historical and philosophical questioning of the emergence, nature and meaning of these structures seems secondary in this tradition.

Secondly, Penelope Maddy's work on the choice of axioms in set theory has called into question one of the consequences of Quine's holism (the non-autonomy of mathematics with respect to physics) and certain aspects of the indispensability argument. She thus opens the way for consideration of the arguments which, within the mathematical community, lead to the rejection or acceptance of certain axioms and certain entities rather than others. However, Maddy argues, in full accordance with Quinian naturalism, that mathematics does not allow for deeper forms of justification than are operational in current mathematics. This way of posing the problem certainly opens up far wider investigations into mathematical practice and methodology than the Quinian horizon, but they remain circumscribed, in Maddy's work, to the realm of set theory and Quinian naturalism.

A final challenge to Quinian dogmas comes from the opening up of epistemology to the history of mathematics, a consequence of the reception of Lakatos' doctrine by analytic philosophy. The notion of "mathematical practice" can receive a clear definition in this tradition, being linked Lakatos's idea of the development of competing research programmes. Consequently, attention to the historical development of knowledge opens the door to considerations related to heuristics, to non-formal or non-logical processes of explanation and argumentation outside set theory, although the empiricist bias of the Lakatos tradition tends to restrict mathematical practice to a *problem-solving* approach.

All these elements have certainly opened up new avenues of analysis that the new wave of the philosophy of mathematical practice is currently developing.

However, as Paolo Mancosu makes clear in his book on the subject, this new wave does not wish to call into question the programme of analytic philosophy, but only to broaden (as far as possible)(19) the field invested by the philosophical analysis of mathematics, without undertaking a clarification of what is meant by mathematical practice.(20) Actually, in full observance of Quinien naturalism, which denies any specificity to philosophical enquiry, this is done by opening up within it to valuable considerations which belong rather to the fields of psychology, sociology or the history of academic institutions. The problem is that without clarifying from the outset the question of what is meant by mathematical practice, and how it fits into the dogma of naturalism, we risk to leave out whole areas of mathematical experience, which concern creation, heuristics, the meaning of the problems posed and the solutions envisaged, leaving it to the cognitive sciences alone to inform us about what these processes involve.

Perhaps this radical questioning and opening up to other perspectives of analysis, which analytic philosophy seems to want to accomplish, could be set in motion by a more attentive analysis of this French tradition in the philosophy of mathematics.

The latter is profoundly anti-naturalist, believing in epistemological ruptures, and at the same time it is fundamentally attentive to present and past mathematical experience. It is profoundly philosophical and even metaphysical, and at the same time aware that nothing can be said about mathematics without a careful analysis of its present, its history, the way in which the singular experience of the constitution of mathematical objects and theories develops over time.

(19) See Brendan Larvor *Review of Paolo Mancosu' the philosophy of mathematical practice*, in *Philosophia Mathematica,* Volume 18, Issue 3, October 2010, Pages 350-360.

(20) This empiricist bias is certainly the opposite of the neo-Kantian conceptions of the tradition we are presenting here. Already in the *Critique of Pure Reason*, Kant expressed himself clearly against the empiricist myth that claims to construct theories by gleaning disparate observations without any prior plan: "They [those who founded modern science] comprehended that reason has insight only into what it itself produces according to its own design; that it must take the lead with principles for its judgments according to constant laws and compel nature to answer its questions, rather than letting nature guide its movements by keeping reason, as it were, in leading-strings; for otherwise accidental observations, made according to no previously designed plan, can never connect up into a necessary law, which is yet what reason seeks and requires." B XIII, translated and edited by P. Guyer, A. Wood, Cambridge University Press.

The distinctive feature of the French tradition in the philosophy of mathematics that we are presenting in this issue is that it places the notion of structure at the heart of its thinking, while at the same time addressing the question of the constitution of mathematical objects, and taking an interest in the concrete practice of mathematics (mathematical experience), without however rejecting out of hand the question of foundations, because as Jean Cavaillès so aptly put it: "I am not trying to define mathematics, but by means of mathematics to understand what it means to know and to think".

It is also profoundly philosophical in the sense that it does not shy away from engaging mathematics with philosophy and its history, without a priori judgements or preconceived ideological exclusions, because what is at stake, as Lautman says, is the questioning of the meaning and scope of mathematical experience. And despite all this, it has not given up on dialogue with mathematicians. This obviously complex dialogue between these philosophers and the mathematicians of their time, this dialogue that they construct over and above all the radical differences in language, methods and even interests, is felt to be a vital necessity for philosophy and is at the foundation of what has often been called the mathematism specific to this tradition.

This shows that attention to mathematical practice, far from excluding the question of foundations, actually enriches it.

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