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A triptych on empirical philosophy of mathematics. Part II: Who?

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Abstract. This is the second part of a tripartite paper discussing the research focus of the CIPSH Chair *Diversity of Mathematical Research Cultures & Practices* (DMRCP). We discuss a philosophical approach called "empirical philosophy of mathematics" and say why it should be done, who should do it, and how it should be done. This second part deals with the question "who?".

§ 1. — Introduction.

The *International Humanities Council* (*Conseil International de Philosophie et des Sciences Humaines*; CIPSH) has established a *CIPSH Chair* entitled *Diversity of Mathematical Research Cultures & Practices* (DMRCP) at the Universität Hamburg. This institution aims to study mathematics as a human discipline using methods from the empirical social sciences and the humanities with a particular focus on the effect of differences in research cultures and practices on mathematics.

While the term "philosophy" is not part of the name of the CIPSH Chair, it stands in the tradition of a wider research community that was or is involved in the conference series *Perspectives on Mathematical Practices* (PMP) (Brussels; 2002 & 2007), the network *Philosophy of Mathematics: Sociological Aspects & Mathematical Practice* (PhiMSAMP) funded by the *Deutsche Forschungsgemeinschaft* (DFG; 2006–2010), as well as the closely related *Association for the Philosophy of Mathematical Practice* (APMP). Like the CIPSH chair, this research community is motivated and driven by questions of philosophical nature about mathematics and studies the discipline of mathematics on the basis of empirical data.(1) For the purpose of this paper, we shall introduce the loosely defined term

empirical philosophy of mathematics

for this particular approach: by *empirical philosophy of mathematics*, we refer to philosophy of mathematics whose philosophical arguments make use of empirical data about mathematical practices, i.e., the way mathematicians speak, act, and collaborate, their institutions and incentive structures, their implicit and explicit value judgments etc.

In our *Triptych* consisting of three closely connected parts (cf. also [25, 26]), we shall argue why it is reasonable to assume that empirical data about mathematical practices is relevant for philosophy of mathematics, discuss whose task it is to provide such empirical data, and give a description of empirical philosophy of mathematics in practice.

(1)Cf. [30] for a discussion of the peculiar relationship between this research community and the field of philosophy.

The type of empirical data. We emphasise that we stipulate that *empirical philosophy of mathematics* specifically uses *empirical data about mathematical practices*, not just any type of empirical data.

This distinction is considerably more salient for other disciplinary philosophies of science than for philosophy of mathematics. For an empirical science, e.g., physics, chemistry, or biology, there are two very different types of empirical data that might be relevant for the philosopher: empirical data about the physical, chemical, or biological world and empirical data about scientific practices in physics, chemistry, or biology.

Only very extreme philosophical positions (e.g., positions that in principle deny the existence of physical reality or our ability to gain empirical information about that physical reality) would reject the view that empirical data about the physical, chemical, or biological world has some bearing on philosophy of physics, chemistry, or biology. Use of such empirical data in philosophical arguments is common in philosophy of science and would not constitute *empirical philosophy of physics, chemistry, or biology* according to our terminology. The use of empirical data about the practices of physicists, chemists, or biologists, on the other hand, would not be accepted by all or possibly even most philosophers of science in philosophical arguments. It is the latter type of empirical philosophy of science that we are focusing on.

Note that this distinction is less straightforward for a non-empirical discipline (such as mathematics): the question whether empirical data has any relevant impact on mathematics at all is a debated philosophical position within philosophy of mathematics. *Empiricism* is a philosophical position about mathematics, taking a particular view of how mathematical knowledge is obtained: an empiricist philosopher of mathematics would claim that empirical knowledge plays an important role in mathematical epistemology.(2) Depending on her philosophical views, an empirical philosopher of mathematics could argue either for or against empiricist philosophy of mathematics on the basis of empirical data about how mathematicians corroborate their knowledge in their research practices. Most importantly, being an empirical philosopher of mathematics does not imply any position on the question of empiricism in mathematics.

(2)For an overview of the early empiricism of John Stuart Mill (1806–1873), cf. [45, § 4.3]; modern empiricist positions usually relate to the Quine-Putnam indispensability argument [6, 42].

CIPSH Chairs. It is reasonable to assume that very few readers know what a CIPSH Chair is, as this is a relatively new programme instituted by the International Humanities Council (*Conseil International de Philosophie et des Sciences Humaines*, CIPSH), modelled after the better known programmes of UNESCO Chairs, in 2018. In spite of their names, neither UNESCO Chairs nor CIPSH Chairs are are professorships, but rather research networks. We quote from the UNESCO documentation:

> A UNESCO Chair is a project and a team at a university or a higher education or research institution that partners with UNESCO in order to advance knowledge and practice in an area that is a priority for both the institution and UNESCO. [46, p. 5]

Similarly, a CIPSH Chair is a network of several research institutions, centred around one institution that created the CIPSH Chair and appointed its chairholder. In addition to the network that constitutes each individual CIPSH Chair, the CIPSH Chair programme creates another higher-level network where the chairholders can exchange experiences and best practices. The CIPSH Chairs programme was proposed and discussed by the 33rd General Assembly of CIPSH in Liège on 6 August 2017 and created by the CIPSH Executive Committee on 16 April 2018 during its meeting in Xiamen. Until early 2024, eight CIPSH Chairs were created, covering topics such as ethnolinguistics, digital humanities, and spiritualism.

DMRCP. *Diversity of Mathematical Research Cultures & Practices* (DMRCP) is one of these eight CIPSH Chairs. It studies cultural phenomena in mathematical research, with a particular emphasis on phenomena where different research cultures diverge on their practices, to obtain a multi-dimensional description of the nature, epistemology, and methodology of mathematical research, contrast it with other disciplines, and describe and understand the cultural variance across various boundaries (e.g., national, disciplinary, or sectorial boundaries). This research must necessarily involve a plethora of disciplines, including (in alphabetic order) anthropology, history of science, mathematics education, philosophy of science, psychology, and sociology, and actively involve research mathematicians.

The goal of the network is to catalyse research cooperation between the network nodes, to coordinate the global research activities in this area, and to provide infrastructure (both for research support and research dissemination) to those researchers working in the area.

The authors of this paper are the scientific coordinator and the chairholder of DMRCP, respectively. DMRCP has both Senior Fellows and Fellows: the Senior Fellows together with the chairholder form the DMRCP Council, the main deliberative body of the network that makes strategic decisions and elects the Fellowship.

The inaugural Senior Fellows were Hanne Andersen, Carolin Antos-Kuby, Helen De Cruz, Karen François, Christian Greiffenhagen, Gerhard Heinzmann, Matthew Inglis, Catherine Jami, Brendan Larvor, Ursula Martin, Helena Mihaljević, Eva Müller-Hill, R. Ramanujam, Dirk Schlimm, and Bart Van Kerkhove.(3) DMRCP Fellowship is open to researchers interested in the aims and scope of DMRCP: readers interested in the network are invited to contact the first author to discuss a nomination for election.

The triptych & this paper. This paper forms the second part of a triptych consisting of three papers in which we shall discuss and partially answer three fundamental questions about the approach of *empirical philosophy of mathematics*: "**Why** should we do empirical philosophy of mathematics?", "**Who** should provide the empirical data for empirical philosophy of mathematics?" and "**How** should we do empirical philosophy of mathematics?" The three parts of the triptych elaborate on and continue the discussion started in the papers [29, 30, 31], respectively.

In Part I of the Triptych [25], we discussed the fact that philosophical concepts have non-philosophical counterpart concepts that are used in ordinary language; in particular, concepts from philosophy of mathematics have counterparts used in the technical language of mathematicians. Statements about these non-philosophical concepts are empirical statements. We argued that the meta-philosophical position that these empirical statements have no bearing whatsoever on our philosophical theory is tantamount to severing the link between the philosophical concepts and

(3) Hanne Andersen resigned from her Senior Fellowship on 1 July 2023 and was replaced by Henrik Kragh Sørensen on the DMRCP Council.

their natural language counterparts entirely. In line with naturalistic positions in philosophy, we consider this to be an undesirable meta-philosophical position.

For this paper, we shall assume that we have convinced the reader that empirical facts about mathematical practices are philosophically relevant and discuss how to obtain these empirical facts: in particular, we wonder **who** is responsible for providing the empirical data for this endeavour. We shall argue that the philosophers of mathematics will have to play an active part in the empirical research themselves.

The answer to the question **Who?** means that empirical philosophy of mathematics is a major effort that requires learning the methods of other disciplines and training our junior researchers in non-philosophical research skills. Part III of the triptych will provide a description of **How** to do empirical philosophy of mathematics and what the active role of the philosopher is.

§ 2. — Empirical data about the natural world vs. empirical data about human practices.

We remind the reader that *empirical philosophy of mathematics* is philosophy of mathematics whose philosophical arguments make use of empirical data about mathematical practices. In § 1, we highlighted the difference between using empirical data about the physical world and using empirical data about human institutions, concepts, and activities.

When philosophers use empirical data about the physical world, they are usually not involved in the empirical work themselves; rather, they rely on empirical research done by scientists independently of the philosophical debates.

To give a number of examples: Firstly, the rich philosophical literature connecting the question of the existence free will to findings in the neurosciences (cf. [38]) relies on the work by many social and cognitive psychologists such as Libet's seminal work on *Bereitschaftspotential* [28] that was done without active involvement of professional philosophers. In the philosophy of mind, discussions about the unity of consciousness, e.g., Parfit's thought experiments on split brains [39], use empirical facts about neurosurgical split-brain procedures which were originally developed in the 1930s, long before philosophers started to think about these problems.[4] Finally, in the philosophy of physics, an extensive literature on philosophical issues concerning quantum mechanics exists [36, 47], most of which is relying on experimental findings of physicists that were done independently of philosophers' work.

We should emphasise that we do not wish to interpret the term "independently" too strictly in this description: notions such as free will, consciousness, personal identity, and the nature of physical reality are philosophical concepts of wider interest to a community extending well beyond the world of academic philosophy, including the scientists who did the empirical work in our examples. As a consequence, it is only natural that the scientists were influenced by their own philosophical views as well as the history of philosophical discussions about the notions. Given our academic practices, it is not unlikely that these scientists interacted with philosophers on funding panels, university committees, in academies, at interdisciplinary conferences, or within the collegiate university, and therefore had the chance to exchange thoughts with philosophers about the philosophical consequences of their work.

However, the causal and temporal relationship in these cases is clear: the scientists did the empirical work without the immediate or primary intention to contribute to the philosophical debate and they did it prior to the active debate in the philosophical literature. To consider the example of Parfit's split brain thought experiment: the brain procedures of *commissurotomy* and *corpus callosotomy* were first used in the 1930s and 1940s with purely therapeutic intentions (avoiding the spread of an epileptic seizure to spread from one brain hemisphere to the other), long before Parfit came up with his thought experiment in the 1970s.

§ 3. — Operationalisation.

In the following, we shall use a particular empirical statement about mathematical practice as a running example that will inform our discussion.

> Given a sufficient amount of time and effort, any two mathematicians will be able to agree on whether a proof in their joint area of expertise is correct and complete. (Φ_1)

[4] Cf. [34, Footnote 18] for a discussion of "medically informed [philosophical] publications".

This statement is commonly used in philosophical debates about the special epistemic status of mathematics(5) and it has been the focus of empirical testing in a series of papers.(6)

This empirical sentence contains several terms that need to be specified before it can be empirically tested, corroborated, or refuted. The concepts "mathematician", "agreement", "area of expertise", and "sufficient amount of time" require *operationalisation* to determine whether a particular data point is a positive or negative case for (Φ_1). E.g., we need a clear empirical condition for whether a test subject is a mathematician, whether a particular proof falls into their area of expertise, and whether the observed behaviour of two mathematicians constitutes agreement.

Note that the necessity of operationalisation is not a special feature of empirical sentences in the social sciences; it is also required for sentences in the natural sciences. Consider the following natural science sentence structurally analogous to (Φ_1).

> Given a sufficiently high increase in temperature, any substance in liquid state will go into the gaseous state. (Φ_2)

The sentence (Φ_2) contains natural language terms such as "liquid state", "gaseous state", and "sufficiently high temperature" that need to be operationalised in order for any experiment to serve as a corroboration or refutation of (Φ_2). While the natural language terms in (Φ_2) are not precise, there are natural interpretations for them that are uniformly accepted meanings for the concepts, except in cases specifically designed to test the limits of our definitions.

In contrast, for concepts dealing with human institutions and practices, there is very little intersubjective standardisation. E.g., there are many different operationalisations of the term "mathematician" that are all eminently reasonable. To give a few examples:

M1 A person who holds a higher education degree in mathematics.

M2 A person who has had research experience in mathematics in the past, as witnessed by at least one research publication.

M3 A person who is currently employed to do mathematical research.

Clearly, **M1**, **M2**, and **M3** do not describe the same group of people; in fact, for any two of them, it is easy to find a person who belongs

(5) For a general introduction into the discussion of the special epistemic status of mathematics, cf. [40, 41] and [32, § 1.1, in particular Footnote 4].

(6) Cf. [24, 49, 50] and the discussion in [31, pp. 28–29].

to the former, but not the latter and vice versa. The differences are large enough that the status of (Φ_1) is likely to depend heavily on which of the operationalisations we choose.

At the same time, for each of the three operationalisations, it is easy to think of contexts in which it is a reasonable operationalisation of the term "mathematician". So, how do we pick the right one?

We shall tackle this question in § 4; before we do so, we should like to make two highly relevant methodological remarks:

First we observe that the level of specificity of our operationalisations provided is not sufficient for actual decisions: taking **M1** as an example, someone who holds a degree called *Bachelor of Science in Mathematics* clearly falls under **M1**, but what about a degree in "Business Mathematics" or "Mathematical Physics" of "Statistics"? In the North American university system, do only graduates who majored in mathematics count, or also those who minored? If the degree is in a language other than English, should we look for degrees that have the dictionary translation of "mathematics" in their name?(7) Details like these have haunted empirical projects doing analyses of mathematics graduates and their career trajectories such as the *Pipeline Project* of the *International Mathematical Union* as it is not clear whether national statistics of people falling under **M1** from different countries measure the same thing [3, 9].

Furthermore, the choice of operationalisation does not just affect the interpretation of the data, but can influence the data itself. Suppose that we are collecting data to test (Φ_1) and conduct a study with test subjects. For each data point, we need to decide whether (according to our chosen operationalisation) the two individuals involved were mathematicians, the result fell into their area of expertise, and whether the they agreed on correctness and completeness after an appropriately long period of discussion. Typically, some of these decisions will be based on asking the test subjects a number of relevant questions. But questions about their status as researcher and area of expertise may very well have

(7) To illustrate the issue underlying the last question, note that "science" is the dictionary translation of the German word "Wissenschaft" (or the Dutch word "wetenschap"), but the disciplines that fall under the heading "Wissenschaft" are not the same as the disciplines that fall under the heading "science" and English does not even have a concise term that would describe the scope of the German term "Wissenschaft".

an effect on their later behaviour in the experiment and therefore change the actual data.

§ 4. — Adequacy of operationalisations.

We have now seen that an empirical statement about mathematical practices such as (Φ_1) is not testable per se, but requires operationalisation; furthermore, the operationalisation is not unique and the choice of the operationalisation affects the result of the test. How does the empirical researcher decide which operationalisation to use?

In the empirical social sciences, we need to return to the original *research intention*: the research project that prompted us to ask the empirical question (Φ_1) had some goal; each of the operationalisations will produce false positives and false negatives with respect to our expectation.[(8)] Whether these false positives and false negatives created by our choice of operationalisation jeopardise our study depends, among other things, on whether the existence of these discrepancies is in conflict with the original intention of the research project. Similarly, at the end of § 3, we mentioned that in practice, getting the information needed for interpreting the data may affect the data; again, whether we are worried by these effects in the experimental set-up can only be determined with the original research intention in mind.

To illustrate this with a medical example: suppose we have a viral test and a test population in which some test subjects are infected with the virus and others are not. We try to determine how good our test is; whether we are more worried about false negatives or false positives depends on the context. If the virus is deadly and there is a cheap drug with no side effects, we should like to have no false negatives and do not mind administering the drug to some people who are not infected; in contrast, if the virus is not particularly dangerous and the drug is outrageously expensive or has major side effects, we should aim to avoid false positives.

[(8)] E.g., a mathematics professor who has a degree in English literature and therefore does not meet the requirements of **M1**, a very prolific and eccentric mathematical researcher who has never published and therefore does not meet the requirements of **M2**, a historian who was hired by a mathematics department and whose contract by administrative default mentions the phrase "mathematical research" placing them within the scope of **M3**, even though they are not expected to do mathematical research.

The design decisions made as part of the experimental methodology must and will reflect this.

In the natural sciences, the difference between acceptable operationalisations tends to be much lower than in the social sciences, and we often find that methodologies are largely independent from the research intentions. This higher degree of intersubjective stability means that empirical results can be used as evidence for different research projects; this is the situation found in the examples of philosophy using empirical data from the natural sciences in § 2.

That is not equally true in the social sciences: an experiment done within a particular framework has incorporated research design decisions based on the research intention of the original empirical researcher which may be incompatible with the research intention of a different researcher who wishes to use these results later. An example of this issue is highlighted in the literature by the meta-discussion about psychological experiments in economics and psychology: experiments in economics routinely identify payoffs in decision situations with financial incentives; that identification is not done in experiments performed by psychologists, even if dealing with very similar research questions.[9]

We make no claims about a general structural difference between the natural and the social sciences: in fact, variations in operationalisation and design decisions based on research intentions exist in the natural sciences as well as in the social sciences. However, we claim that the degrees of variance and dependency are considerably higher in the social sciences than they are in the natural science.[10]

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(9) Cf. [2, 5, 48]. Hertwig and Ortmann explicitly discuss this: "Empirical tests of theories depend crucially on the methodological decisions researchers make in designing and implementing the test. (p. 383) ... [E]xperimental economics and corresponding areas in psychology (in particular, behavioral decision making) have very different conceptions of good experimentation. (p. 384) ... [T]he most important argument motivating financial incentives, is that most economics experiments test economic theory, which provides a comparatively unified framework built on maximization assumptions (of utility, profit, revenue, etc.) and defines standards of optimal behavior. Thus, economic theory lends itself to straightforward translations into experiments employing financial incentives. ... Psychologists typically do not rely on a similarly unified theoretical framework that can be easily translated into experimental design. (p. 390)" [18]

(10) We cannot back our claim up with numerical data: doing so would be an interesting research question for a methodologist of science or a philosopher of social sciences.

This phenomenon will play an important role in our argument in § 5.

§ 5. — The sartorial metaphor.

In [30], the second author introduced a *sartorial metaphor* for the uses of empirical data in philosophy. The empirical work in philosophy as mentioned in § 2 is described as

> using empirical data like ready-to-wear or off-the-rack clothes, provided on the shelves and racks of a store with no direct input by the customer expressing his or her preferences of clothing style or fit; on the other end of the spectrum, one would have the bespoke experience where the customer can determine every detail of the garment and the garment is then tailored exactly to the specifications of the customer and made to fit his or her body perfectly. Those who have the appropriate skills could even become the tailors themselves, getting rid of any need to communicate wishes and desires. Bespoke empirical philosophy would be a project in which the philosopher works very closely with the empirical scientist and designs an experiment or other observational activity jointly with her or him; the extreme case of bespoke would be do-it-yourself where the philosopher becomes an empirical scientist and does the empirical work herself or himself. Of course, the more extravagant and non-standard your desires and wishes with respect to your clothes are, the less likely it is that you will find these off the rack and you might have to move towards bespoke tailoring. [30, p. 36]

As was argued in [30], the easiest approach for the empirical philosopher interested in an empirical statement such as (Φ_1) is to search the library for empirical studies that provide answers to (Φ_1) that have already been provided by empirical researchers, i.e., *off-the-rack* empirical philosophy.[11] If these studies do not exist

[11] This approach avoids a situation where the philosopher has to do empirical work themselves, reducing both the involved effort and also the risk of inadequate empirical data, as pointed out in [30, p. 37]: "[I]f you find a high-quality garment off the rack that fits you very well, it may be considerable better than a bespoke

(yet), they can either approach empirical researchers and ask them whether they would like to do these studies for the philosophers ("*bespoke*") or do the studies themselves ("*do-it-yourself*").

For empirical philosophy of mathematics, we first observe that empirical data about mathematical research practices is rare: while the area of *Science and Technology Studies* developed in the 1970s and 1980s provided empirical data about research practices in the natural and technological sciences, mathematics has been considered by this community only rarely. In the main monograph on the sociology of mathematics [17], Heintz comments "[d]ie Soziologie [begegnet] der Mathematik mit einer eigentümlichen Mischung aus Devotion und Desinteresse" (p. 9); after Heintz's seminal book came out, a number of papers have been published,(12) but compared to the sociology of other sciences, the literature is still very sparse. The literature of cognitive science and psychology on number cognition is very rich,(13) but relatively few cognitive scientists deal with research practices in mathematics.(14)

However, the discussion in § 4 shows that the problem is not merely the dearth of sociological and cognitive science studies dealing with questions such as (Φ_1): even if they existed, the sociologists' and cognitive scientists' choice of operationalisation (and therefore methodology of collecting the data) would have been based on their research intentions and consequently unlikely to give the empirical philosopher precisely what they need for their philosophical argument. This combination of the general lack of empirical research about mathematical research practices and the close relationship of the small number of existing results with the research intention of the original author means that any successful use of off-the-rack empirical data in empirical philosophy of mathematics would be a serendipitous exception to the rule.

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garment from a mediocre tailor or (in the case of most of us with no expertise in tailoring) a garment that you made yourself. Similarly, in empirical philosophy, there are advantages and disadvantages to both ready-to-wear and bespoke approaches. In the first instance, philosophers are not empirical scientists, so do-it-yourself empirical philosophy requires that the philosopher acquire the skills and learn the techniques of another discipline, wasting time and energy that could be spent on something that they are more qualified for (such as doing philosophy) and possibly even leading to sub-standard or flawed empirical work."

(12) Cf., e.g., [11, 12, 13, 14, 15, 16, 33, 43].

(13) Cf., e.g., [1, 4, 7, 8, 10, 27, 37].

(14) A notable exception is the work by Matthew Inglis and his collaborators, including a number of empirical philosophers of mathematics; cf. [19, 20, 21, 22, 23, 35, 44] and also the references in Footnote (6).

Without overstretching the metaphor, we can say that in tailoring as well as in empirical philosophy, there is a spectrum between ready-to-wear and bespoke. Not every problem with style or fit or every desirable detail that cannot be found in off-the-rack clothes requires a fully bespoke order. A more affordable option is *alterations*, either done by an alterations tailor or the customer themselves. Letting out or taking in, changing a button, or adding simple design features can be done with a substantially lower degree of tailoring expertise than required for full do-it-yourself tailoring. Similarly, not every study used in empirical philosophy of mathematics needs to be designed from scratch; instead, the literature can provide studies that would be useful after alterations: e.g., a sociological study of a different discipline can be modified and replicated for the case of mathematics. We believe that the future of empirical philosophy of mathematics lies in collaborations between philosophers and empirical researchers as well as the acquisition of enough empirical research skills by the philosophers to do the necessary alterations required to adapt and perform existing studies to the philosophical context.

§ 6. — Conclusion.

We have argued that in the particular case of empirical philosophy of mathematics, off-the-rack empirical data is hard to find. This means that empirical philosophers of mathematics will either need to request bespoke studies from empirical scientists or do their own empirical work.

Contractual research is common in the applied sciences; it is the model closest to bespoke tailoring in our sartorial metaphor: a research team that lacks expertise for doing some particular task hires another research team to perform that task for them. However, empirical studies cost time and money and it is unlikely that philosophers, in particular philosophers of mathematics, have the financial means to afford paying empirical scientists to do specific research tasks for them.

As a consequence, bespoke empirical philosophy of mathematics is more likely to consist of collaborations where philosophers and empirical scientists work together on a project and become coauthors of the resulting publications. This also comes at a cost: in order to convince the empirical researcher to join the project, the research questions have to be of interest to them and therefore might require some modification in the form of compromises. Furthermore, the philosophers have to become familiar enough with the methods of empirical research to contribute to the methodological decisions and whether they are in line with the original research intention.

If the empirical philosopher cannot find empirical scientists to collaborate with or their expectations of compromise are not acceptable to the philosopher, the philosopher will have to become an empirical researcher herself, either by altering studies from the social science literature or by designing new studies. As a consequence, the answer to our question "**Who** should provide the empirical data for empirical philosophy of mathematics?" is that the philosophers have to be actively involved in the collection of empirical data.

By and large, philosophers are not trained for this and training a new generation of empirical philosophers of mathematics to have the expertise to either contribute to or even lead empirical research is a major additional effort that is necessary to do empirical philosophy of mathematics successfully. The CIPSH Chair DMRCP aims to play its role in this effort by coordinating training activities for junior researchers in the field.

Embarking in the project of training a new generation of philosophers in additional skills is a major commitment; in Part III of our Triptych [26], we follow a paradigmatic example to highlight how empirical philosophy of mathematics is done.

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