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The epistemology of the mathematical "dedans" in Albert Lautman's early writings

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These days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain

Hermann Weyl

Abstract: The youthful writings of Albert Lautman are examined, in particular the Rapport Bouglé of 1935, where the major conceptual nucleus of his subsequent research path aimed at entering 'inside' the contents of the sciences are already identified; it is no coincidence that, from the beginning, the philosophical effort has been directed to understand on the one hand the singularity of mathematics and on the other, to clarify 'les enjeux' of the close connection between mathematics and physics, 'l'unité physico-mathématique'. And all this finds its reasons in Lautman's having been a faithful interpreter of Hermann Weyl and of a certain Hilbert; and developing some of their points has allowed him to give an autonomous contribution to the philosophie mathématique, a chapter of epistemological thought produced in the French-speaking area, unique in its kind and still little known.

Keyword: Lautman, Weyl, Hilbert, Epistemology, philosophie mathématique, Philosophy of physical Mathematics

§ 1. — From Maximilien Winter to Albert Lautman.

Sometimes, to better understand the career of little-known figures and the genesis of some of their ideas that have not found their rightful place in the critical literature, it is more profitable to look at minor writings produced on various occasions, as in the case of Albert Lautman (1908-1944), writings that we take into consideration in our contribution. These texts already identify some of the key points of his discourse and must be integrated into the analysis of the important indications present in the *Bouglé Report*(1) , which is strategic for understanding the subsequent results. Some of these were papers read at two important cultural events at European level, such as the two *Congrès de Philosophie Scientifique* held in Paris in 1935 and 1937, which showed full inclusion in the ongoing debates in the field of the philosophy of science.(2) Although brief,

(1)It was thanks to Fernando Zalamea's research at the ENS *Archives* in Paris that he discovered the "Rapport sur les travaux philosophiques entrepris par M. Lautman" of March 1935, submitted by the young Lautman for a research project; this manuscript was physically found by Jacques Lautman at the *Archives Nationales, Fonds ENS, Bouglé, Code 61 A.J. Carton 96* and transcribed by Zalamea himself in *Philosophiques*, 37/1, (2010), pp. 9-15 and for an initial analysis of this *Report*, transcribed by Zalamea himself, cf. "Mixtes et passage du local au global chez Lautman : préfiguration de la théorie des faisceaux", *ibid*, p. 17-25.

(2)These congresses were decided on at the VIII$^{\text{th}}$ International Congress of Philosophy, held in Prague in 1934, because of the importance of the philosophy of science at the beginning of the XX$^{\text{twentieth}}$ century and the decisive role played by the protagonists of the Vienna Circle. It should be borne in mind that this line of research of an epistemological nature had already taken shape with a plurality of perspectives from the first International Congress of Philosophy held in Paris in 1900 thanks to the editors of the *Revue de Métaphysique et de Morale.* During this event, there were major debates, particularly on the nature of mathematics, thanks also to the presence of Henri Poincaré, Bertrand Russell and David Hilbert, and on the strategic role played by this review in the first decade of the century, cf. the various contributions published in issue 2014/4, no. 84 entitled "L'Europe philosophique des congrès à la guerre" and especially that of S. Soulé, "La *Revue de Métaphysique et de Morale* et les Congrès Internationaux de Philosophie (1900-1914): une contribution à la construction d'une Internationale philosophique", pp. 467-481. It should be remembered that the 1937 edition was devoted to Descartes, whose philosophy was considered by the organisers to be the first serious and organic attempt at 'scientific philosophy' that the new scientific philosophy of the XX$^{\text{twentieth}}$ century had to face up to. Although the term 'philosophy of science' is generally associated with that in use since the Vienna Circle, its origins lie in the positivist climate and it was Ernest Renan who used it in a text published in 1890, but written several decades earlier, cf. E. Renan, *L'avenir de la science*, ed. by A. Petit, Paris, Garnier-Flammarion, 1995, p. 301.

these writings by Albert Lautman[(3)] heralded a different way of approaching epistemological problems from that which was emerging within the framework of this new discipline, which had already established itself autonomously; but this still young knowledge often offered, in his eyes, "a disappointing spectacle" to certain unilateralisms that it encountered, and in particular philosophical reflection on mathematics. And it was hoped that it would be necessary "to wish philosophy of science a higher ambition".[(4)] In the same *Rapport Bouglé*, the foundations were laid for a path aimed at going beyond "the traditional discussion between formalists and intuitionists" to grasp "the internal structure of mathematical edifices rather than giving certain categories such as numbers an unjustifiable primacy"; and the young Lautman was already assigning a particular task to philosophical work, aimed in particular at highlighting "the rational connection between the whole and its parts.... to see the reasons for this logical connection, one of the roots of mathematical truth, and we see in it only one of the fundamental principles that philosophical criticism can derive from the study of mathematical theories".[(5)]

These writings thus announce and enucleate certain conceptual points expressed later in a more organic way in the works of 1937, such as *the Essay on the notions of structure and existence*

[(3)] They can be found in A. Lautman, *Les mathématiques, les idées et le réel physique*, Paris, Vrin, 2006: "Considérations sur la logique mathématique" (1933, pp. 39-46); "Mathématiques et réalité" (Communication au Congrès International de Philosophie Scientifique, Paris 1935, pp. 47-50); Compte rendu du "Congrès International de Philosophie des Sciences, 15-23 septembre 1935" (pp. 51-64); "De la réalité inhérente aux théories mathématiques" (Communication au IX^e Congrès International de Philosophie, Paris 1937, pp 65-68) and "L'axiomatique et la méthode de division" (1937, pp. 69-80). This different attitude may explain why, in the words of Jean Petitot, "such an inspired mind could be so little celebrated", cf. J. Petitot, "Refaire le Timée. Introduction à la philosophie mathématique d'Albert Lautman, *Revue d'Histoire des Sciences*, t. 40, (1987), p. 113.

[(4)] A. Lautman, "Congrès International de Philosophie des Sciences", p. 64; this essay, published in 1936 in the *Revue de Métaphysique et de Morale,* is not simply a chronicle of an important cultural event, but turns out to be a close critical confrontation with the non-homogeneous positions taken by the adherents of the Vienna Circle School, starting with R. Carnap; the distance is taken by analysing the contributions of Gödel, Tarski, Bernays and C. Chevalley. These latter "Hilbertian mathematicians [have]... taken up Hilbertian theory on new bases" to show "to the great plaisir of philosophers and the surprise of logicians, that there was something else in metaphysics than the famous pseudo-problems"; they also make it possible to "rediscover in mathematical thought the effort of the human person to insert into automatisms everything that has ceased to be life and real needs", *ibid.*, p. 55.

[(5)] A. Lautman, *Bouglé Report*, p. 10.

in mathematics and *the Essay on* the *unity of the mathematical sciences in their current development*, followed in 1939 by a short but dense and significant text such as *New research on the dialectical structure of mathematics*[6]. Taken as a whole, they highlight the various philosophical-scientific sources that nourish them and are already programmatically aimed at understanding the particular nature of mathematics as a producer of real "knowledge"; they examine the particular cognitive processes considered as the result of "mathematical reason" in the process of being made, and of a "mathematics that is alive and in action"[7] and with its own history, which must be the object of the philosopher's attention in particular, as his master Léon Brunschvicg rightly emphasised.

Aimed at a better understanding of the reality of mathematics, Lautman's approach is characterised by the fact that "you have to go inside", as Simone Weil first said in one of her many references to mathematical thought[8] and then Jean Desanti said of Jean Cavaillès,

[6] The first two constitute the doctoral thesis and the supplementary thesis; it should be remembered that this 1939 work, as well as the posthumous 1946 work entitled *Symètrie et dissymétrie en mathématiques et en physique. Le problème du temps*, which appeared in a series of Hermann "Essais philosophiques" edited by Jean Cavaillès and Raymond Aron, a series in which Lautman himself collaborated in the drafting; cf. also the other important posthumous work, *La pensée mathématique, the* fruit of a debate with Cavaillès at a Séance de la Société française de Philosophie, held in Paris in 1939 (the latter translated into italian in our *Alle origini della 'nuova epistemologia'. Il Congrès Descartes del 1937,* Lecce, Il Protagora, 1992, pp. 149-171). Almost all of Lautman's works were first published in 1977 and reprinted in the new edition of 2006, an edition included in the English translation, *Mathematics, Ideas and the Physical Real,* translated by Simon B. Duffy, London/New York, Continuum International Publishing, 2011. Fernando Zalamea has edited the Spanish translation which, compared with the French translation, is complete, entitled *Ensayos sobre la dialéctica, e structura y unidad de las matemáticas modernas,* Bogotà, Universidad Nacional de Colombia, 2011; this edition contains previously unpublished material and also reviews of Lautman's works such as Paul Bernays's 1940 review and a related bibliography.

[7] A. Lichnerowicz, "Albert Lautman et la philosophie mathématique", *Revue de Métaphysique et de Morale, t.* LXXXIII, 1, (1978), pp. 24-32 and p. 25 ; and our " Les mathématiques et l'expérience selon Albert Lautman", in E. Barbin-J.P. Cléro (eds.), *Les mathématiques et l'expèrience*, Paris, Hermann, 2015, pp. 311-338 ; "Introduzione" to A. Lautman, *La matematica come resistenza*, translated into italian, post-fazione of Fernando Zalamea, Roma, Castelvecchi, 2017, pp. 7-43 and "For an epistemology of mathematical contents: Albert Lautman", *Lettera Matematica*, International edition, Springer, pp. 1-10.

[8] S. Weil, *Œuvres complètes*, VI. I-4 "Cahiers", Paris, Gallimard, 1994, p. 94. Simone Weil, André's sister, like Lautman, attended Brunschvicg's classes at the École normale supérieure in Paris with her thesis on Descartes; in her writings, almost all posthumous, there are numerous references to Greek mathematics and to the work of

who "made us go inside with him. He did not set out a 'philosophy of mathematics' that would have provided an external view of the object".[9] It is no coincidence that, as in the French epistemological tradition, Cavaillès was already stressing the fact that "for the philosopher, more than for anyone else, theories that have been tried out and barely sketched out are as fruitful as definitive results".[10] At the same time, Lautman began to put in place certain elements,developed in his later work, which see mathematical formalisms as a living organism that is not just "the set of propositions derived from axioms, but organised, structured, complete *êtres*, having as it were their own anatomy and physiology".[11]

Galois, Argand, Gauss, Riemann and Hilbert; and on this aspect, cf. our *Razionalismi senza dogmi. Per una epistemologia della fisica matematica*, Soveria Mannelli, Rubbettino Ed, 2004, ch. IV; L. Lafforgue, "Simone Weil et les mathématiques" in E. Gabellieri-F. L'Yvonnet (eds.), *Simone Weil. Cahiers de l'Herne*, Paris, Édition de l'Herne, 2014, pp. 126-137 and F. Zalamea, "Géométrie, Topologie, Riemann, et les nuances vivantes de la pensée mathématique chez Bachelard (avec un contrepoint autour de Simone Weil), *Bachelard Studies*, (2022), 1, pp. 69-78. We can also take it in the sense expressed by Hourya Benis-Sinaceur, who took it from an "image borrowed from Henri Michaux" in *Corps et Modèles. Essai sur l'histoire de l'algèbre réelle* (Paris, Vrin, 1999), in order to include a "fundamental aspect of the change in the relationship between mathematics and logic" (*ibid*., pp. 20-21); cf. the recent and significant text in honour of Hourya Benis-Sinaceur, which has been compared, among others, with the work of Jean Cavaillès and Lautman, cf. E. Haffner-D. Rabouin (dir.), *L'Épistémologie du dedans. Mélanges en l'honneur de Hourya Benis-Sinaceur*, Paris, Garnier, 2021.

(9) J. T. Desanti, "Introduction" to J. Cavaillès, *Méthode axiomatique e formalisme*, Paris, Hermann, 19812, p. 7. This is how we might define Desanti's own path, which he set out in *Les idéalités mathématiques* in 1968 and then in the writings collected in 1975, *La philosophie silencieuse ou critique des philosophies des sciences.* As in *Mathésis, idéalité et historicité,* ed. established by D. Wittman with preface by D. Pradelle, Paris, ENS Éditions, 2014; and on this figure cf. our *Epistemologia debole*, Verona, Bertani, 1985, ch. II and "Jean Desanti: per una teoria delle idealità matematiche", *Bollettino di Storia della filosofia dell'Università di Lecce*, vol. XI, (1993-1995), pp. 247-258; G. Ravis-Giordani (ed.), *Jean Toussaint Desanti, une pensée et son site*, Paris, Rue d'ULM-ENS, 1997 and D. Pradelle-F. Sebbah, *Penser avec Desanti*, Paris, TER, 2010.

(10) A. Lautman, "Considerations sur la logique mathématique", p. 46. Already in this short essay, the results of the debates on the foundations of mathematics are analysed together with the related attempts to define the "mathematical *êtres* by logisticians and intuitionists"; they distance themselves from these attempts, which were considered very instructive for philosophical work, as Kurt Gödel would later say in almost identical terms in the writings of the 1950s contained in his *Nachlass*. Some of Jacques Herbrand's ideas were developed that enabled him to read Hilbert differently, already projected towards this interpretation based on the notion of structure, which was later clarified, also thanks to his contacts with the nascent group of Bourbakists.

(11) A. Lautman, "Mathématiques et réalité", p. 48; a vision of mathematics as a "living organism" is already present in the last pages of *Problemi della scienza*

This aspect, the attention paid to the dynamic moments in which the "mathematical *êtres*" are formed and not just to the moments of their logical stability, is a central theme in this unique current of *mathematical philosophy* that developed in France and in particular after the advent of non-Euclidean geometries and with the recognition of the philosophical dimension of the works of Galois[(12)]. And in this cultural area more than in others, thanks to the critical confrontation with the "geometer-thinkers" or "philosopher-scholars" from Clifford to Riemann and Poincaré[(13)], particular figures of "philosopher-scholars" were created, committed to clarifying the crucial transition "from the absolute to the relative"[(14)] in the various sciences. To use Bourbaki's famous metaphor[(15)], the

of 1906, a work later translated into French in two volumes (1909 and 1913). Federigo Enriques was a figure who had maintained close relations with the French-speaking epistemological community since the first decade of the last century; his various works received varying degrees of attention from Gaston Bachelard, the young Cavaillès, who in several of his works spoke of the "living fabric of mathematics", Lautman, Gonseth, Piaget and Hélène Metzger. This is why, in our various studies, we have spoken of an Italian-French neo-rationalist epistemology, as well as of 'hidden harmonies' as Ch. Alunni has spoken of in various writings and especially in *Spectres de Bachelard. Gaston Bachelard et l'école surrationaliste*, Paris, Hermann, 2018.

(12) See É. Galois, *Œuvres mathématiques*, with an introduction by É. Picard, Paris, Gauthiers-Villars, 1897; P. Dupuy, *La vie d'Évariste Galois*, (1896), Paris, Cahiers de la Quizaine, 1903 with a preface by J. Tannery who, in *the Bulletin des Sciences Mathématiques*, 30, (1907), edited the publication of some manuscripts.

(13) It was the Italian mathematician Federigo Enriques who introduced this expression in *Problemi della scienza*, referring to Riemann, Grassmann and Helmholtz; the expression "savants-philosophes" to designate scientists such as Mach, Hertz and Maxwell was later put forward by Harald Höffding in *Philosophes contemporains*, translated from the German, Paris, F. Alcan. Alcan, 1908^2, pp. 95-98 and on the role of mathematician-philosophers, see also P. Parrochia, *Mathematics and Philosophy*, London-Hoboken, ISTE Ltd- J. Wiley, 2018, part 1 and part 4.

(14) Cf. J.-C. Pont, "De l'absolu au relatif, destin du XIX^{e} siècle", in J.C. Pont et al. Pont et *al*, *Pour comprendre le XIX^{e}. Histoire et philosophie des sciences à la fin du siècle,* Firenze, Leo S. Olschki Ed. 2007, pp. IX-XLVIII. Figures such as Brunschvicg, Milhaud, Meyerson, Le Roy, Winter, Couturat, A. Rey and Boutroux, authors of the "founding texts of French epistemology", are cited in A. Brenner (ed.), *Les textes fondateurs de l'épistémologie française,* Paris, Hermann, 2015. These figures, along with the 'savants-philosophes' such as Duhem, Berthelot and others of the second half of the XIX$^{\text{nineteenth}}$ century, gave rise to a rich research tradition and on this cf. S. Bordoni, *When Historiography met Epistemology. Sophisticated Histories Philosophy of Science in French-speaking Countries in the Second Half of the Nineteenth Century*, Leiden/Boston, Brill, 2017.

(15) See N. Bourbaki, *Eléments d'histoire des mathématiques*, Paris, Hermann, 1969, p. 27.

aim of the various protagonists was to overcome this pathological vision of mathematical truth produced by the development of non-Euclidean geometries; and at the same time, all efforts were directed towards understanding the true meaning of the epistemic fracture they produced in the philosophical field by examining the growing processes of abstraction and ”progressive conceptualisation”, as Bernhard Riemann called them in his philosophical fragments.(16) Philosophy of mathematics can be seen as a genuine tradition of research in the epistemological field, responding to this need and grasping the new characteristics that what was called ”mathematical reality” was taking on, and which can be seen in the undertaking led by the editors of the *Revue de Métaphysique et de Morale*. A new conceptual space was opened up and focused, at least in the early years, on the close link between philosophical reflection and the changes taking place in the mathematical corpus, as well as on the need to address the question of ”what philosophy for what mathematics?”.(17)

(16)Cf. B. Riemann, ”Erkenntnisstheoretisches” in *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*, ed. by R. Dedekind and H. Weber, Leipzig, Teubner, 1876, pp. 521-525; some of Riemann's writings were translated by G. Darboux into *Bulletin des Sciences Mathématiques* in 1872, then reprinted in 1898 with a preface by F. Klein. But this field of philosophical-scientific research has its roots in the Cartesian season and in the reflections of the Encyclopaedists, to the point of distinguishing another era in particular, as Léon Brunschvicg puts it, the positivist period with a ”mathematical philosophy... dominated by the mathematical work of Lagrange”, cf. L. Brunschvicg, *Les étapes de la philosophie mathématique*, (1912), preface by J. Desanti, Paris, Blanchard, 1972, p. 292-301. Between the end of the nineteenth century and the beginning of the twentieth century, another 'stage' took place that could be described as 'post-positivist', thanks to the presence of various figures such as Brunschvicg, mentioned above, in comparison with the works of Riemann and Felix Klein's Erlangen Programme; the term 'mathematical reason' is present in Brunschvicg's aforementioned work, in the various works of Gaston Milhaud and Edouard Le Roy, and above all in Gaston Darboux's *Éloges académiques et discours* de 1912 (Paris, Hermann), a text later taken up by Kurt Gödel. To understand the specificity of French-speaking mathematical philosophy, it should be remembered that it was almost alone in dealing with the philosophical significance of Riemann's work, as was the case for Bachelard and Lautman, also thanks to the mediation of Hermann Weyl ; Riemann, in fact, had little consideration in the philosophy of science of the early XIX[twentieth] century, as Ludovico Geymonat stated in *Filosofia e filosofia della scienza*, Milano, Feltrinelli, 1960, I appendix, and on the Riemannian roots of French-speaking epistemology, cf. our *Razionalismi senza dogmi*, op. cit, ch. I and ch. III.

(17)Cf. J.P. Marquis- F. Patras, ”Preface” to vol. 1, n. 1 of the *Annals of Mathematics and Philosophy* 2023); and on the role of the *Revue de Métaphysique et de Morale*, ”une pourvoyeuse de sciences (1893-1947)” and a veritable ”conceptual laboratory

But in order to understand the path taken by Lautman, it may be useful to consider the fact that, in this context, the commitment of a completely forgotten figure, such as Maximilien Winter (1871-1935), played a decisive role; this particular figure came to be situated among both mathematicians and philosophers, in order to be as faithful as possible to the activity of mathematicians by highlighting their particular needs for truth and rigour, cognitive values considered useful in the philosophical field. His path has been one of continuously straddling the frontiers between the various fields of mathematics and strategically placing himself at their crossroads in order to understand their conceptual transformations; and he was already oriented towards grasping implicit thought from the inside with a specific historical-critical methodology borrowed from that of Ernst Mach for physical theories. Outside academic circles, he concentrated his writings on the ″philosophical importance of number theory″, the ″characteristics of modern algebra″, the ″logical introduction to the theory of functions″, the ″role of philosophy in scientific discovery″, ″intuition in mathematics″, and the relationship between ″metaphysics and mathematical logic″. He set out to give more theoretical substance to germinal epistemology, to use Bachelardian terminology that was present in the work of mathematicians of the time, from Du Bois-Reymond to Klein, Borel and Poincaré, and to compare the results of what was then called logistics[(18)] ; and this choice led him to distance himself from Léon Brunschvicg himself, who also, in his 1912 book *Les étapes de la philosophie mathématique*, confronted dynamic movements within the mathematics building in the second half of the XIX[nineteenth] century. But in Winter's eyes, his path was still within the theory of knowledge of a classical stamp, since he limited himself to describing the interplay of concepts from the outside; Brunschvicg was, in fact, an outstanding ″philosopher-reader of mathematics already

and eminent product″ of the École normale supérieure de Paris, cf. Ch. Alunni, *Spectres de Bachelard*, op. cit. p. 267 and Appendix I, Appendix II.

(18) Cf. M. Winter, *La méthode dans la philosophie des mathématiques*, Paris, F. Alcan, 1911, where some of his writings published between 1905 and 1910 in the *Revue de Métaphysique et de Morale*, of which he was one of the founders, are collected with slight modifications; and on one of the first works on this figure, cf. Ch. Alunni, ″Maximilien Winter et Federigo Enriques : des harmonies exhumées″ (2015), in *Spectre de Bachelard*, op. cit. pp. 259-287. We have translated some of these writings into Italian with a long introduction, cf. ″Il contributo di Maximilien Winter alla *critique des sciences*″, in M. Winter, *Il metodo storico-critico per una nuova filosofia delle matematiche*, Milano, Meltemi Ed., 2020, pp. 9-85.

done", as Jean Desanti noted in 1974 on the occasion of the republication of his major work, and then Fréderic Patras, considering his philosophy as a "descriptive mathematical philosophy... historically important, but no longer sufficient today".[19]

Winter already considered that such a commitment was insufficient in his day because of the increasing complexity attained by mathematical thought, also following the advent of mathematical logic, which in his view in any case brought with it a change of perspective in the relationship between mathematics and logic that needed to be critically evaluated[20] ; This event was studied by examining typical problems of an epistemological nature, such as the major questions on methods, intuition, rigour and formalism, through the analysis of the notions of number, function, group and continuous variable, thanks to a comparison with the work of Borel and Lebesgue. It was for this reason that he felt the need to "draw from the purely scientific work of M. Poincaré, arguments in favour of true philosophy, which is none other than the spirit of science itself".[21] In his writings, Winter uses Bachelardian *ante litteram* language and borrows it from A.-A. Cournot, who already felt the need to take into serious consideration only those "renewed crises of the sciences" useful for the renewal of philosophical reflection[22] ; he thus considered it strategic to work on a new stage of mathematical philosophy by situating at the "avampostes" of mathematical thought for a real turn in this field. And we could say

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[19] J. Desanti, "Préface" to L. Brunschvicg, *Les étapes de la philosophie mathématique*, op. cit, p. V and F. Patras, *La pensée mathématique contemporaine*, Paris, PUF, 2001, p. 167.

[20] M. Winter, "Métaphysique et Logique mathématique", *Revue de Métaphysique et de Morale*, t. XIII, (1905), pp. 589-619. Winter confronted logistics, considered as a non-secondary event, and interpreted it as one of the various methods of approaching certain questions without arriving at foundational points of view in the sense of Louis Couturat; and for an overall idea of the role of this figure, cf. many of his writings contained in L. Couturat, *Mathématique, langage, philosophie*, Paris, Garnier, 2017 and the proceedings of an international colloquium held in 1977, cf. M. Loi et *al.*, *Louis Couturat... de Leibniz à Russell*, Paris, ENS Rue d'Ulm, 1983. Lautman himself said on several occasions that "mathematical logic enjoys no special privilege in this respect; it is only one theory among others and the problems it raises or solves are almost identical elsewhere", cf. "De la réalité inhérente aux théories mathématiques", cit. p. 66.

[21] *Ibid*, p. 611.

[22] A-A. Cournot, *Essai sur les fondements de nos connaissances et sur les caractères de la critique philosophique*, Paris, Hachette, 1851, t. I-II, t. I, p. 23.

of Winter[23] , without any exaggeration, what Bachelard himself said in 1938 in a letter to Lautman after reading his first works: "by ousting calculations, you have managed to keep thought... Every scientific philosopher knows that this is an almost insurmountable difficulty... With what sympathy, I salute in you a representative of the young team that is going to bring philosophy back to the heroic tasks of difficult thought. If only there were a dozen of us".[24]

And in his writings, as in those of Lautman, we notice this uncommon attitude of thought in outlining the contours of a philosophical reflection more closely linked to the specific contents of the corpus of mathematics, Winter thus being one of the first of this hoped-for "dozen" to understand the origin of "new entities", and "how mathematical theories grow and are transformed...". We won't dwell on the formal developments in calculus, but will focus on highlighting ideas"[25]; and it is no coincidence that he set out to bring about the "heroic" emergence of scientific thought and, above all, the constitutive role of concepts through the "chain of ideas in the sciences", following in Cournot's footsteps.[26] And in this context, but in a hidden way, it is no coincidence that Winter was one of the organisers and the indispensable collaborator of Xavier Léon in the organisation of the first and only 'Congrès International de Philosophie Mathématique', which took place in Paris shortly before the outbreak of the Great War, with objectives based on a

(23)To understand Winter's stature, we need to bear in mind some of the judgements of various mathematicians, such as Maurice Fréchet, who described his work, on the occasion of his death, on "the movement of ideas", which led to the development of the functional calculus, as "a masterly study", *Revue de Métaphysique et de Morale*, t. XCV, (1935), p. 1. For Jacques Hadamard, Winter is "one of the philosophers who has the best understanding of scientific subjects" in *Essai sur la psychologie de l'invention dans le domaine mathématique*, (1945), Paris, Librairie Blanchard, 1959, p. 86. On a more philosophical level, Brunschvicg often cited him in *Les étapes de la philosophie mathématique*, op. cit. pp. VII-VIII and pp. 545, 551, 553 and 555; it is strange that Gaston Bachelard never mentioned him.

(24)G. Bachelard in H. Benis-Sinaceur, "Lettre inédite de Gaston Bachelard à Albert Lautman", *Revue d'histoire des sciences*, t. XL, 1, (1987), p. 129.

(25)M. Winter, "Caractères de l'algèbre moderne", *Revue de Métaphysique* et de Morale, vol. XVII, (1910), pp. 496-497 and see also "Du rôle de la philosophie dans la découverte scientifique", vol. XVI, (1908), pp. 901-920.

(26)Cf. A. A. Cournot, *Traité de l'enchaînement des idées dans les sciences et l'histoire,* Paris, Hachette, 1861. In "À propos d'une nouvelle conception de la philosophie des sciences", *Revue de Métaphysique et de Morale,* t. II. (1894) pp. 606-621, a young Winter sees in Cournot a concrete way of making mathematical objects play a role in several other contexts: sometimes geometric, sometimes physical and sometimes economic; and, to use Lautman's terms, the fact that they are "mixed".

post-brunschvicgian perspective in mathematical philosophy[27] . This Congress was the first attempt, then interrupted because of the First World War, to give life to specific sectors of the nascent philosophy of science, to a genuine epistemic community at international level capable of dealing with the philosophical problems inherent in mathematical reality in all its singularity and complexity; It is therefore no coincidence that Lautman used this term in a minor work and that, in many of his writings, he adopted several of Winter's concerns, such as that of assigning to 'philosophical criticism' the task of grasping the links between the different sectors of mathematical thought without absolutizing one to the detriment of the other.[28]

(27) This can be said because since the first International Congresses of Philosophy, they have been organised by the editors of the *Revue de Métaphysique et de Morale*, and Winter, on Léon's behalf, has been responsible for contacts with the various European schools of mathematics. This information is thanks to Charles Alunni, who searched the *Correspondence Xavier Léon* (1868-1935). Xavier Léon manuscripts.

(28) It was a strategic point that emerged, thanks to Winter's role, in the first and only "Congrès de Philosophie Mahématique", held in Paris in April 1914 after the first four International Congresses of Philosophy from 1900 to 1911. This Congress was sponsored by the *Revue de Métaphysique et de Morale*, by the Société Française de Philosophie with the collaboration of the *Encyklopädie der mathematischen Wissenschaften* in preparing the philosophical part; and this is due to the commitment of the Italian mathematician-epistemologist Federigo Enriques (1871-1946) who prepared the statutes of what was to become a new International Society of Mathematical Philosophy, a fact emphasised by the editors of the *Revue* by presenting only the address of the President Émile Boutroux, "Congrès International de Philosophie Mathématique", *Revue de Métaphysique et de Morale*, t. XXII, no. 5, (1914), pp. 571-580. Some of these reports were subsequently published in different editions of the journal rather than in a single issue, as had been planned. Cf. A. Reymond, "Chronique. Le premier Congrès de Philosophie Mathématique", L'*Enseignement mathématique*, 16 (1914), pp. 370-378. We believe that with this Congress, which is the subject of our current research, at least in the French-speaking world, a new "stage in mathematical philosophy" began, thanks to the non-secondary role played by Winter who, with Enriques, succeeded in bringing together Whitehead with only the two Germans L. Nelson and D. König, the French Hadamard, Couturat, Le Roy, Brunschvicg, the Italian A. Padoa and others such as A. Reymond, author of a text on mathematical logic taken up by Lautman in his first minor work.

§ 2. — Albert Lautman, Hermann Weyl's faithful interpreter.

Lautman's epistemological commitment is situated on this already articulated background, already rich in points of view[29] and strengthened by a critical awareness of the continuing development of the corpus of mathematics that took place in the 1920s and 1930s, launched increasingly "on the rational conquest of complexity" in the words of Gaston Bachelard[30] , in order to grasp the various changes at work with other, more appropriate conceptual tools[31] ; In fact, mathematics was undergoing other, more profound upheavals "at the furthest reaches of its granite empire" and contained such "new characteristics" that they required a "reversal" of the existing perspective, as Hermann Weyl had already indicated in *Das Continuum* in 1918.[32]

[29] For Ch. Alunni, "Winter is an essential philosophical reference for Albert Lautman, at strategic points in his argument", such as "the two occurrences, the ascent towards the Absolute and the 'Schémas de genèse", in *Spectres de Bachelard*, pp. cit., pp. 270-271.

[30] G. Bachelard, *Études sur l'évolution d'un problème de physique. La propagation thermique dans les solides,* (1928) with a preface by A. Lichnerowicz, Paris, Vrin, 1973, p. 163 ; and on a recent critical examination of Bachelard's conception of mathematics, see the contributions by Frédéric Patras, Sandra Pravica and Fernando Zalamea in *Bachelard Studies*, 2 (2022).

[31] In the same years, Ferdinand Gonseth's mathematical philosophy moved in this direction at the Zurich School, where Hermann Weyl and Paul Bernays also landed. The author of several books, including *Les fondements de la mathématique* (1928), *Mathématique et réalité* (1936) and *Philosophie mathématique* (1939); in 1938 Gonseth organised first Conversation on the Structure and Method of Mathematics in the Light of the Results of Weyl, Gödel, Heyting and Gentzen, with the aim of building a new philosophical platform for analysing *the "creation of ideas"* (cf. F. Gonseth, "Sur la doctrine préalable des vérités élémentaires", in *Les Entretiens de Zürich sur les fondements et la méthode des sciences mathématiques,* Zürich, Leeman, 1941), p. 16; and on this Conversation, to which Gödel and Gentzen were also invited but who, for various reasons, did not attend, see our *Federigo Enriques e la 'nuova epistemologia'*, Lecce-Brescia, Pensa Multimedia-ENS 'Pensée des sciences', with introduction by F. Zalamea, 2019, Part II, ch. IV.

[32] Cf. H. Weyl, *Le continu et autres écrits*, french translation by J. Largeault, Paris, Vrin, 1994, *passim* e *Temps, espace, matière*, French translation by G. Juvet and R. Leroy, Paris, Librairie Blanchard, 1922, p. 1-2. And it is no coincidence that Lautman's minor writings confronted Gustave Juvet's *La structure des nouvelles théories physiques* (1933), in which a structural vision of mathematics was put forward thanks to Hermann Weyl's group theory; and cf. A. Lautman, "Congrès International de Philosophie des Sciences", pp. 62-64 and on Juvet in relation to Weyl and Lautman, cf. Ch. Alunni, "Gustave Juvet, (1896-1936), un pionnier oublié", in *Spectres de Bachelard,* op. cit, pp. 209-257.

It is no coincidence that this figure was at the centre of his interests when he examined group theory and the fundamental work on Riemann and saw in it a real ″consciousness of axiomatic thought[(33)]″, as Federigo Enriques called it, to be extended to different fields; In this way, Lautman, thanks also to his knowledge of the algebraic work of Emil Artin and Emmy Noether, came face to face with the ″El Dorado of the axiomatic method″, seeing it as one of the tools ″of concrete mathematical research″, having enabled the development of abstract algebra, which is essentially the theory of algebraic structures.[(34)]

But what is considered to be the most strategic is the Weylian contribution to a better understanding of the fact that the axiomatic method is immanent to mathematical objects in certain situations and capable of better highlighting the structures that make generalisations possible and the structural aspect of the theorems themselves; Lautman's path is therefore critically situated on the crest of the relationship that, in the 1930s, was established between ″the angel of topology und the devil of abstract algebra″[(35)] and the new philosophical issues that these disciplines were able to raise. And the aim of all this was to grasp the philosophical consequences, as Maximilien Winter had done for the algebraic theories and number theory of his time, thus sketching out ″the dialectical structure of mathematics″; this is how Lautman situates his path of mathematical philosophy in a genuine philosophy of ″mixed mathematics in act″ linked to the ″ideas and notions that make it possible to understand the passage of these potentially mixed mathematics″.[(36)] But to guide him through the inevitable but necessary philosophical shifts and to link him, for example, to the Platonic philosophy of mixed mathematics, as in ″Axiomatics and the method of division″, Weyl's indications in *Raum, Zeit, Materie*

(33) Cf. F. Enriques, *L'évolution de la logique,* french translation by G.. Monod-Herzen, Paris, Chiron, 1926, *passim*; and on the analogies and convergences between Enriques and Weyl, cf. our ″Hermann Weyl et Federigo Enriques. Philosophie et mathématiques″, in Ch. Alunni et *al.* (eds.), *Albert Einstein et Hermann Weyl (1955-2005). Questions épistémologiques ouvertes,* Manduria, Barbieri-Selvaggi Ed. ENS Éditions Rue d'Ulm, 2010, p. 69-87.

(34) H. Weyl, ″Emmy Noether″, (1935) in *Gesammelte Abhandlungen,* III, Berlin-Heidelberg-New York, Springer-Verlag, 1968, p. 438 and ″Topologie and abstrakte Algebra als zwei Wege des mathematischen Verständnisses″, (1932), *ibid*, p. 349; for the development of real algebra, cf. E. Artin-O. Schreier, ″Algebraische Konstruktion reller Körper″, *Abh. Math. Sem. Hamburg*, 5, (1926), pp. 85-99 and on this important chapter, cf. H. Benis-Sinaceur, *Corps et Modèles*, op. cit.

(35) H. Weyl, ″Invariants″, *Duke Mathematical Journal*, 5, (1939), p. 500.

(36) F. Zalamea, ″Lautman and the creative dialectic of mathematics″, cit. p. 23.

remain; in the first pages of this work, it is deemed strategic to take into consideration the origins and "beginnings" of mathematical thought, considered "obscure". It is precisely the mathematician who, in his highly developed science, operates with his notions, who needs to remember that the origins go back further than his methods show. The first thing to do is to seek to understand; despite the fluctuations of philosophy and its oscillations from system to system, we must not give up this search, if knowledge is not to be transformed into incomprehensible chaos".(37)

The constant confrontation with Weyl's theses was useful for Lautman, on the one hand, to engage in a broader philosophical strategy and, on the other hand, to free himself from the theorisations of the founding schools, while tackling typical themes such as the existence of mathematical beings; at the same time, his approach to mathematics, seen in light of the "particular problem of the whole and its parts", enabled him to see in the Hilbertian conception of non-contradiction the important fact that "mathematical logic is so rich in philosophical consequences in the hands of Hilbert and how ungrateful it has seemed until now in the hands of those who have seen in it only a grammar".(38) The figure of Hermann Weyl enabled him to rethink and broaden the concept of rationality in mathematics and to have more tools at his disposal to access mathematical *êtres* that cannot be circumscribed in purely formal domains, and to see them as the result of connections between different notions and not limited to one of them such as number. And many of his analyses in these minor writings(39) , like those in Gaston Bachelard's *Essai sur la connaissance approchée*, can be seen as responses to the "philosophical considerations" put forward by Hermann Weyl in several of his works(40) ; they aimed to see mathematics not as "mere play", because then, in the name of "their security", it would be totally removed from the universal history of the mind. We must therefore try to ensure that mathematics is in some way given a role in the service of knowledge".(41)

(37) H. Weyl, *Temps, espace, matière*, op. cit. p. 8; the terms 'glissements' and 'fluctuations de la pensée philosophique' are also found in the writings of Maximilien Winter.

(38) A. Lautman, *Bouglé Report*, op. cit. p. 10.

(39) Cf. A. Lautman, "Mathématiques et réalité", p. 48 and especially in the first pages of the *Essai sur l'unité des sciences mathématiques dans leur développement actuel*.

(40) H. Weyl, *Temps, espace, matière*, op. cit. p. 2.

(41) H. Weyl, "Les degrés de l'infini" (1931), in *Le continu et autres écrits*, op. cit. p. 305; and on this cf. C. Eckes, "Les sources mathématiques d'Albert Lautman", in E. Haffner-D. Rabouin (eds.), *L'épistémologie du dedans*, op. cit. p. 427-431.

This is why, from the outset and not by chance, his career, like that of Winter, was not limited to the study of "mathematicians with wings", but took into serious consideration those who had "wings", as Henri Poincaré called them, with other strategic indications aimed in particular at better perceiving their "weight", in the sense of attributing a strategic philosophical relevance to the changes underway[(42)] ; They were sketched out in the famous 1908 essay "L'avenir des mathématiques": "The true method is to study their history and their present state" also because "mathematical science must reflect on itself". From the outset, epistemological work had to aim to understand that mathematics is a creative product of the human mind because of its specific ability to take fewer elements from the outside world and to be the primary conceptual tool for exploring it.[(43)] And already, from these minor writings, it is very clear that Lautman's aim was to clarify this fact philosophically, also because he believed that the philosophies of mathematics of his time did not give the appropriate epistemic weight to this problem and to the fact that a mathematical being could take on different configurations under the weight of conceptual transformations. In this way, mathematics undergoes "extensions which, in the qualitative sense, bring interruptions" within itself, and acquires greater meaning, as Federigo Enriques and Gaston Bachelard[(44)] affirmed in unison thanks to their critical confrontation with "the geometer-thinker" Riemann; In the memoirs of this authentic mathematician 'with wings', we see the juxtaposition of points of view that are far removed from one another, such as the processes of geometrization of algebraic functions, the introduction of differential elements into geometry, the role of analytic functions, the relationship between geometry and physics - processes that turn mathematics into thought. They are therefore genuine "events of conceptualisation", of mathematical reason, where "the dynamic history of thought is written" and

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(42) This is how we understand the term "weigh" in its close relationship with "think", as Fernando Zalamea illustrates in *Modelos en haces para el pensiamento matemático,* Bogotá, Universidad Nacional de Colombia, 2021, pp. 48-52, in the wake of his work on Lautman and Grothendieck.

(43) H. Poincaré, "L'avenir des mathématiques", *Scientia. Rivista di scienza*, a. 2, III, (1908), p. 1 and p. 15.

(44) F. Enriques, *Les problèmes de la science et la logique*, trans. franç. par J. Dubois, Paris, F. Alcan, 1909, p. 77 and G. Bachelard, *Le nouvel esprit scientifique*, Paris, PUF, 1971[11], p. 56.

the result of their intrinsic mobility with different stakes and virtuality's, in the words of Bachelard and Gilles Châtelet.[(45)]

And a new 'stage of mathematical philosophy' was to respond to these problems in order to better understand 'the art of mathematics', a 'great art with inexhaustible resources', as the founders of the *Revue de Métaphysique et de Morale* called it in 1893 in the editorial of its first issue; Thus mathematical thought, strengthened by the epistemic acquisition of its own historical-conceptual dimension, is increasingly becoming a major source for philosophy itself, which it regards as its "elder sister"[(46)] because it in turn aims to give meaning to the reasons for reality. And like others in this journal such as Winter, Lautman saw in the work of mathematicians with 'wings' an implicit philosophical meaning that it was not common to develop; and he was not afraid to reconcile himself with a certain Platonism that was seen to be latent in it, as in the case of Kurt Gödel's incompleteness theorems[(47)] . For the same reasons, he then confronted Heidegger himself: the notions of essence and

[(45)] *Ibid.* and cf. G. Châtelet, *Les enjeux du mobile. Mathématique, physique, philosophie*, Paris, Le Seuil, 1993 and *L'enchantement du virtuel. Mathématique, physique, philosophie*, ed. by Ch. Alunni and C. Paoletti. Paoletti, Paris, Éditions Rue d'Ulm, 2010. G. G. Granger, who has also written about Cavaillès and Lautman, has spoken of mathematical virtualities between 'determinism and freedom' in various works such as *Philosophes en Liberté*, Paris, Ellipses, 2001 and *Science et réalité*, Paris, Odile Jacob, 2001; there is a special section on mathematical philosophy in his *Essai d'une philosophie du style*, Paris, Odile Jacob, 1988^2. Nowadays there is an aptitude for thinking in "transitive" mathematical philosophy in the sense put forward by René Guitart in "Deux problèmes en vue d'une épistémologie transitive des mathématiques", *Revue de synthèse*, t. 136, n. 1-2, (2015), pp. 237-279 where certain theses by A. Grothendieck and F. Patras.

[(46)] Cf. "Introduction", *Revue de Métaphysique et de Morale, t. 1,* (1893), p. 1; and for a recent reinterpretation of this aspect, cf. G. Lolli-F. S. Tortoriello (eds.), *L'arte di pensare. Matematiche e filosofia*, Turin, UTET, 2020.

[(47)] The clarification of the meaning of the Platonism invoked by Lautman has been the subject of various interpretations, such as that of Jean Petitot already mentioned; but perhaps the 'phenomenological Platonism' proposed by Houria Benis-Sineceur in 'Le platonisme phénoménologique d'Albert Lautman', *Philosophiques*, 37(1), (2010), pp. 27-54. is more significant. On the interpretation of Lautman's incompleteness theorems and on the reasons for the 'Platonic' choice of certain figures such as Paul Bernays, cf. our *Il dibattito in area francofona sul pensiero matematico e Kurt Gödel*, with a preface by L. Magnani and an afterword by F. Patras, Rome, Ed. Studium ebook, 2021. But Halévy, one of the founders of the *Revue de Métaphysique et de Morale*, had already seen in Plato's vision a heuristic aspect for making philosophico-scientific thought mobile in *Théorie platonicienne des sciences*, Paris, F. Alcan, 1896, p. 203 et *passim.*

existence, conveyed in mathematical theories, are considered capable of offering ″a solution quite different from those of intuitionism or formalism″ and not of ″confusing mathematical philosophy with the study of different logical formalisms″.(48) In this way, as is clear from the presentation at the first Congress of Scientific Philosophy in 1935, he shares one of the fundamental ideas of any sound philosophy of science put forward by Moritz Schlick in the preface to the first edition the *Allgemeine Erkenntnislehre* of 1918, in which he clearly states that ″the philosophical element is inherent in all sciences as their true soul, by virtue of which they alone are properly sciences.... Philosophy therefore dwells in the depths of all the sciences, but in not all of them is it equally ready to reveal itself″.(49)

According to Lautman, there were still several ′edifices′ of mathematical thought that had not received adequate consideration of their specific philosophical depth, of their weight in the theoretical sphere, such as abstract algebra, group theory with its implications for physics ; These chapters of mathematical thought, ″so rich in results and so harmonious in their structures″, are still considered to be ″enclosed by the principle of identity″, and the philosopher's task is to ″move away from such poor conceptions and find within mathematics a reality that fully satisfies the expectations he has of them″.(50) And it was also in a minor essay that shortly preceded his first organic work that Lautman sketched out part of his research programme centred on

> ... the problem of genesis, where the transition between essence and existence takes place... which is linked, moreover, to the problem of the finite and the infinite...

(48) 43 A. Lautman, ″De la réalité inhérente aux théories mathématiques″, p. 65.

(49) M. Schlick, *Teoria generale della conoscenza*, transl it., Milano, F. Angeli, 1986, p. 11; such Schlickian ideas (soul of science, philosophy implicit in science) were part of the French epistemological language, as they had been before, for example, with Boutroux, Milhaud, Le Roy, Brunschvicg and then Bachelard. It is also interesting to note that in giving a detailed account of the 1935 Congress, Lautman dwells on certain points of view considered to be common to the positions of Schlick, Brunschvicg and Enriques, whose *Problemi della scienza* had also been translated into German in 1910 with a review in 1911 by Schlick himself; Enriques had been the subject of a session in 1934, on the occasion of the publication of *La signification de l'histoire de la pensée scientifique*, at the Société Française de Philosophie, in which Lautman himself took part along with others such as Jean Cavaillès, Gaston Bachelard and Hélène Metzger, and, with two introductory reports, he took part in the 'Congrès de Philosophie Scientifique' in 1935 and 1937.

(50) A. Lautman, ″De la réalité inhérente aux théories mathématiques″, cit., p. 65.

> a problem of classical metaphysics... This problem is found in mathematics in the discussions concerning the transfinite and the axiom of choice, and intuitionist or formalist mathematicians have generally placed the debate on the terrain of traditional philosophy.(51)

In this way, he traces a path where the questions typical of any philosophical commitment articulated around the objective and truthful values of mathematics(52) are tackled with the critical awareness of an approach more articulated in relation to the positions of his time ; and for this reason we can use Jules Tannery's expression that he was not a "short-sighted philosopher" in understanding the role of mathematics "which can bring order and sequence to our knowledge; they themselves have an order and a logical sequence of their own, which must be discovered by focusing solely on them. Those who can do this will always be rare.(53) There is another question that made Lautman a "rare" figure in his time, committed to building his theoretical edifice as a humble "worker of thought" in sense Maximilien Winter gave to himself(54) ; in his early writings there are constant indications to avoid making an important scientific result into an idol that can lead to "transforming into a scholastic" and its followers into positions of an absolute nature that the same development of the sciences comes to call into question. We can, therefore, speak of another "occurrence" that permeates his path and finds its root in Winter's own effort: we must avoid in any case "transforming into a scholasticism" the limited philosophical interpretations of a theory because any "metascientific extension of a positive doctrine is illegitimate and raises insoluble antinomies".(55)

(51) *Ibid*, p. 66 and 67.

(52) For a comparison with later developments on these themes, see *Philosophie des mathématiques. Ontologie, vérité et fondements*, texts collected by S. Gandon and I. Smadja, Paris, Vrin, 2013.

(53) J. Tannery, "Préface" to P. Dupuy, *La vie d'Évariste Galois*, Paris, Gauthier-Villars, 1903, p. 6; Tannery was referring to philosophers unable to grasp the philosophical meaning of Galois' work.

(54) M. Winter, "Métaphysique et logique mathématique", cit. p. 617.

(55) M. Winter, "Sur l'introduction logique à la théorie des fonctions", *Revue de Métaphysique et de Morale*, t. XV, (1907), p. 187 and cf. note 29.

§ 3. — On another ’effort of thought’: the interpenetration of mathematics and physics.

And the first step to be taken all the way was Lautman’s to give an account in the philosophical field of the scope of axiomatic thinking and its advantages in other fields such as physics, as well as to distance himself from Carnap’s positions already in these minor writings ; but this position *against* Carnap is explained by the fact that Lautman made full use of many other sources drawn from critical confrontation with the rich ”panorama of monographs used” and by having been a ”direct witness to the activities of the mathematics seminar (known as the Julia seminar)” with the establishment of strict ”links and thematic kinships with the Julia seminar papers”.(56) And the minor writings already reflect this total immersion in the ”numerous, repeated and varied borrowings” from the files of this seminar, the ”master source [that] nourished his philosophical reflections... An entire section of Lautman’s philosophy is thus developed from ”(57) this choice to find in certain mathematical theories the tools to understand the ”fundamental passage between existence and essence”, the relationship between ”patterns of structure and patterns of genesis”. And this rich heritage could not fail to lead Lautman to lay the foundations for a ”change of focus in the philosophies of mathematics, his reflections thus aiming to identify the structure of mathematical theories, instead of going back to the foundational elements of which they are constituted”.(58) To this end, unlike most of the

(56)C. Eckes, ”Les sources mathématiques d’Albert Lautman”, cit. p. 431-446. The sessions of this seminar took place from 1933 to 1938; and on the fringes of this seminar, the founders of Nicolas Bourbaki’s group began to meet thanks to André Weil, ”the little Weil” so called because of his young age and his mathematical skills identified as early as 1927 by Maximilien Winter. Other meetings were held in Weil’s house, sometimes attended by his sister Simone, who did not share what he called the ”philosophical disengagement” that initially characterised the Bourbakist group, as can be seen from the various letters the two brothers exchanged on ”the art of mathematics”, cf. S. Weil, *Œuvres complètes*, ”Correspondances”, t VII. I, op. cit. It should be remembered that Lautman and Simone Weil, who had known each other since their days at the École normale supérieure, took part in seminars on Plato in a convent in Marseille during the Occupation; cf. S. Petrement, *La vie de Simone Weil*, voll. I-II, Paris, Fayard, 1973, *passim.*

(57)C. Eckes, ”Les sources mathématiques d’Albert Lautman”, cit. p. 443.

(58)*Ibid*, p. 431; this is why it is more useful to use the expression ”philosophie mathématique” rather than ”philosophy of mathematics”, which belongs to another research tradition. Secondly, even though in many recent writings in the debates of his time, he also turned his attention to other crucial questions concerning the structure of mathematics, in order to gain a better understanding of its real content and the many nuances it brings into play, by having it implement a "living effort of thought" and aiming to construct a genuine "*thought of relation*"[(59)] , to use Gaston Bachelard's expressions; and this is already apparent in one of his little writings, in which he shifts the focus of his interests to the relationship between "mathematics, ideas and physical reality",[(60)] as he himself had already foreseen in 1935 when he drew up the research project contained in the *Bouglé Report*. In fact, he very clearly sketched out the main coordinates of this later problem as he moved away from the most fashionable positions of his time, still in the wake of Hermann Weyl's contributions; these results are seen through the mediation of Élie Cartan's work on differential geometry and generalised spaces with his way of reading and generalising Einstein's theory of gravitation. Both at the beginning of the *Bouglé Report* and at the end, Lautman insisted that

> The most important problem in the philosophy of science is undoubtedly that of the relationship between mathematical theory and physical experience. We would like one day to be able to show how realistic conceptions of the physical universe are merely concrete representations of notions that can only be defined within a mathematical theory. This is certainly the case for the notions of state of a system, energy, discontinuous solutions, continuous spectra and periodicity, which form the basis of contemporary physics...

French-speaking world, "philosophie mathématique" and " pensée mathématique " are used almost synonymously, it is perhaps more useful not to consider them as such, since the specific task of a philosophy of mathematics is to grasp mathematical science as thought, as knowledge tout court, through the identification of its two souls, in M. Schlick's sense, the theoretical and the historical, in an overall vision that would otherwise be lost.

(59) G. Bachelard, *Le rationalisme appliqué,* Paris, P.U. F., 1970^4, p. 214 and p. 208 (underlined by Bachelard).

(60) We use in a methodological sense the title that Fernando Zalamea wanted to give to the 2006 French edition, because it captures very well the general philosophical approach of Lautman's approach, which is shared by some of his interpreters; see also J. Petitot, "Idéalité mathématique et réalité objective. Approche transcendantale", in *Hommage à Jean-Toussaint Desanti*, ed. by H. Benis-Sinaceur, Mauvezin, Éd. T.E.R., 1991, pp. 213-282; F. Zalamea, "Estudio introductorio" a A. Lautman, *Ensayos sobre la dialéctica...*, op. cit, pp. 13-74 and Ch. Alunni, *Spectres de Bachelard*, op. cit,

> We propose to apply the above considerations about the structure of a whole and its parts to physical theories: the notion of a physical system is in fact a global notion that can be defined using the prime integrals and integral invariants of a system of differential equations.[61]

Lautman already envisages this ″capital problem″ in the philosophy of science of the early twentieth century, since, in the purest Bachelardian style, he is able to understand its full epistemic significance, capable of giving ″the scientific mind such complexity, such new characters and aptitudes that all the debates have to be taken up again if we really want to know the philosophical values of science″[62] ; and this problem must be seen in the creation and undoing of fluctuations in philosophical systems in the Weylian sense, because they pose the crucial problem of the relationship of mathematics to reality. And so it becomes necessary for Lautman to ″understand these profound harmonies that can exist between a schematic structure and a material realisation″[63] following the work of Albert Einstein, who made operative Riemann’s indications concerning the way in which mathematics adapts to the different levels of reality in general relativity; and once again Weyl’s indications in group theory are decisive, leading him to take this problem in all its complexity through the mediation of Gustave Juvet’s reading in *La structure des nouvelles théories physiques*, with its insistence on the ″solidarity that manifests itself″ in different domains:

The problems of logic are thus linked not only to mathematical reality, but also to physical reality. M. Juvet, focusing on the problems of group theory, has shown how the study of the structure of a group makes it possible to unite the formal point of view and the concrete point of view in all branches of mathematics.[64]

The task of philosophical analysis is to try to understand how this solidarity manifests itself from within, where close links exist between heterogeneous notions such as Riemann surfaces, groups, numbers, functions and other constructions such as those on different types of space (the structure of space-time, vector space); in this context, increasingly abstract hypotheses such as the tensor calculus play an increasingly strategic role, coming to think physical reality in its various articulations by creating true ′mixtures′; this term is also

[61] A. Lautman, *Bouglé Report*, p. 9 and p. 15.
[62] G. Bachelard, *Le rationalisme appliqué*, op. cit. p. 209.
[63] A. Lautman, « Congrès International de Philosophie des sciences », p. 63.
[64] *Ibid*

derived from his particular reading of Hilbert, seen from a more global point of view also thanks to his basic Kantian baggage present in the philosophical culture beyond the Alps and with the full awareness of the fact that as a whole, from mathematics to physics, "science [has] reached its critical stage".(65) To use Fernando Zalamea's expression(66) , this vision of a mathematics that is "contaminated" at the basis of its thought tends almost by its very nature to create a "mixed", to constitute itself as a "thought of relation"; And Lautman thus manages to give relative weight to questions concerning the structure of theoretical physics by examining the same results as Hilbert and Weyl, to the point of reproaching the adherents of the Vienna School for not having taken them into consideration, even though they are rich in new perspectives and "contents"(67) , by reassigning to Hilbert's *Beweistheorie* itself a different function and extension. Once again, Lautman proves to be a faithful interpreter of Weyl, who in 1926 had very clearly highlighted the constitutive role of mathematics in making the world intelligible:

> A truly realistic mathematics should be conceived, in line with physics, as a branch of the theoretical construction of the one real word, and should adopt the same sober and cautious attitude toward hypothetic extensions of its foundations as is exhibited by physics.(68)

(65) H. Benis-Sinaceur, "Axiomatique et philosophie" in E. Haffner-D. Rabouin (ed.), *L'épistémologie du dedans*, op. cit., p. 518; this articulate essay by Benis-Sinaceur is also very useful for understanding Hilbert's undertaking and Lautman's own reading of it. The presence of Kant and the uses made of him in the French canon in recent years have been the subject of several critical studies, cf. L. Fedi, *Kant, une passion française, 1795-1940*, Olms, Hildeshein, 2018; C. Braveman, *Kant, épistémologue français du XIX^{e} siècle* : *réalisme et rationalisme chez les savants*, Paris, Garnier, 2020 and with particular reference to Brunschvicg, cf. P. Terzi, *La philosophie française au miroir de Kant* (*1854-1986*), Paris, Honoré Champions, 2023.

(66) Cf. F. Zalamea, *Philosophie synthétique de la mathématique contemporaine*, op. cit. chapter II.

(67) Cf. A. Lautman, "Congrès de Philosophie scientifique", cit. Among other things, Lautman took account of certain indications put forward by Hilbert in his report to the Königsberg conference in 1930 entitled *Naturerkennen und Logik*; but Lautman could not have been aware of other Hilbertian writings such as *Natur und mathematisches Erkennen* (1919), published only in 1992 (Basel, Birkhäuser Verlag). And in Lautmanian spirit, we could say that such unpublished writings may have contributed to the misunderstanding of Hilbertian thought on the part of the protagonists of the Vienna Circle; and it is likely that the history of the philosophy of science of the first half of the twentieth century would have been very different.

(68) H. Weyl, *Philosophy of mathematics and natural science*, Princeton, University Press, 1963, p. 235.

But it is still this figure that allows Lautman to see in what Hilbert called "theoretical physics" the consistency of the "real contents" of the sciences, where the crucial question of the truth and objectivity of theoretical constructs is constantly raised, a question felt in all its epistemic force by Ludwig Boltzmann.(69) Like Poincaré, Weyl also wrote in the context of physics as a 'neighbour' of mathematics, and against the new and old empiricist visions of science:

> But Hilbert expressly refers to the neighbouring scientific discipline of theoretical physics. Its hypotheses and laws do not in themselves have a meaning that can be immediately fulfilled intuitively when taken in isolation; it is not the propositions of physics considered in isolation that must in principle be confronted with experience, but only the theoretical structure as a totality. This science does not aim for intuitive knowledge of singular or general states of affairs, nor a *description* that faithfully reproduces the given, but a theoretical, and ultimately purely symbolic, *construction* of the world... It is a profound philosophical question to know what is the nature of the 'truth' or objectivity of such a theoretical construction of the world that goes so far beyond the given.(70)

That's why it's so important

In light of the new results, all Lautman's efforts were directed towards understanding the physical-mathematical unity, since "the constitution of mathematical physics gives us access to reality through knowledge of the structure with which it is endowed"; for this reason, the problem that he considered unduly neglected by much of the mathematical philosophy of his time was constantly under consideration

(69)Cf. L. Boltzmann, "Über die Methoden der theoretischen Physik" (1892), in *Populäre Schriften,* Leipzig, J. A. Barth, 1905, pp. 1-10 and on the creative and inventive role of mathematics in relation to reality for such a scientist, cf. E. Bellone, *Il mondo di carta. Richerche sulla seconda rivoluzione scientifica*, Milano, Mondadori, 1976. For a better idea of the debate on physical theories in Viennese culture at the time, see D. Donato, *I fisici della Grande Vienna*, Florence, Le Lettere, 2011.

(70)H. Weyl, "Remarques et discussion à propos du second exposé de Hilbert sur les fondements des mathématiques" (1928), in J. Largeault, *Intuitionnisme et théorie de la démonstration*, op. cit. pp. 168-169. Suzanne Bachelard spoke of physico-mathematical theories and 'theoretical totalities' in *La conscience de la rationalité,* Paris, PUF, 1958, and on this subject cf. ns. *Rationalismes sans dogmes*, op. cit, ch. V.

> The real philosophical problem is how differential geometry can become a theory of gravitation. This agreement between geometry and physics is proof of the intelligibility of the universe. It results from the mind's development of a way of structuring the universe in profound harmony with the nature of that universe. Understandably, this penetration of reality by the human mind makes no sense to some excessive formalists.(71)

This crucial problem emerged in the outposts of twentieth-century scientific thought, and Lautman found it evident in Hermann Weyl and other figures of mathematical thought in the 1930s; these results lead to "another, more hidden history, made for the philosopher" and aimed at capturing "the dialectical action [that] is constantly playing out in the background"(72) to account for the objective character of theories. In so doing, he redefines the figure of the "mathematical philosopher", or rather the *physical-mathematical philosopher*, who, endowed by his rich conceptual history with more appropriate hermeneutical tools such as the "dialectic"(73) , is seen as more capable of developing a qualitatively different approach to the state of progress of research. As a philosopher of science, and in this very close to the intentions of supporters of the Vienna Circle such as Schlick, as well as those of Gaston Bachelard, Lautman felt almost obliged to bring out this new theoretical 'sense' implicit in the scientific thought of the 1930s, to seek its fundamental unity and to understand this 'dialectical' interpenetration between mathematics and physics on the basis of group theory. to grasp the qualitative leap on a more

(71) A. Lautman, "Mathématique et réalité", cit. p. 100. 49-50.

(72) A. Lautman, *Essai sur les notions de structure et d'existence en mathématiques,* op. cit, p. 131; Lautman thus fully grasps what D. Ria has called *L'unità fisico-matematica nel pensiero epistemologico di Hermann Weyl,* Tesi di Dottorato Univ. del Salento, Galatina, Congedo Ed., 2005, with a preface by F. Patras.

(73) Cf. A. Lautman, *Nouvelles recherches sur la structure dialectique des mathématiques*. The term 'dialectic', even if it refers to the philosophical tradition, is to be understood here in the different meanings assumed in the French epistemology of the 1930s and that of Gaston Bachelard in particular: historical, creative, global, synoptic, multiarticulated, complex, transitive, encompassing or transformative 'synthesis', where contrary concepts or dialectical pairs such as local/global, essence/existence, real/abstract, symmetry/dissymmetry interpenetrate to produce new levels of 'reality' that are always mathematically made possible.

general philosophical level and to highlight the different dimensions and articulations; all this seemed necessary to him and his commitment was therefore aimed at laying the foundations of a broader historical-conceptual perspective that could be described as "synthetic"[(74)] in the sense that it was assumed in France with the orientation of subsequent work towards the primary task of "attempting this synthesis".[(75)] And this is the basic idea behind the *philosophy of physics and mathematics*, which is at the very heart of French epistemological culture[(76)] , to which Lautman made a decisive contribution by offering the most appropriate tools for responding to the revolutions in mathematics and physics, which must therefore be taken together as a *unicum*. In the words of Ludovico Geymonat, he understood better than most the fact that "there was not a revolution in physics on the one hand and in mathematics on the other (set theory, relativity), but a revolution in mathematical physics" as a whole; and, in fact, "the French school was able to recognise the importance of mathematical physics, which is precisely both physics and mathematics".[(77)]

The study of the tensor calculus, Cartan's conception of space, theories of abstract algebra, the theory of Lie groups based on non-commutativity and Pfaff's theory have enabled him to go to the

[(74)] What has been called "the aptitude for synthesis", "the search for synthesis", has characterised a large part of French culture since the beginning of the XX[twentieth] century, from concrete research in the historical and anthropological field to philosophical and epistemological research; genuine movements for synthesis in various fields of research have arisen from an anti-positivist and anti-reductionist perspective, without which it is impossible to understand most French cultural events of the XX[twentieth] century. But similar movements were present in various European countries, and Enriques himself in Italy, founding the journal *Scientia* in 1907, saw it as an instrument of 'scientific synthesis'. The term "synthesis" should not be understood simply as a synthesis of the knowledge produced, but as the profound essence of all science as a tool for grasping the unity of knowledge itself in its full autonomy and diversity, capable then of offering tools at a deeper level to arrive at an unfragmented vision of reality. The figure of the historian Henri Berr, founder of the 'Centre International de Synthèse' in 1925, stood out in this respect, and on this central figure in the French milieu, see E. Castelli Gattinara, *Strane alleanze. Storici, filosofi e scienziati nel Novecento*, Milano, Mimesis, 2003, chapters I-III.

[(75)] A. Lautman, *Essai sur les notions de structure et d'existence en mathématiques*, op. cit. p. 129.

[(76)] Even in the second half of the [twentieth] century, this interest continued, particularly in the works of Gilles Châtelet, already mentioned.

[(77)] L. Geymonat , "Tre domande per Ludovico Geymonat", *Due culture a confronto: la filosofia della scienza in Francia e in Italia nel Novecento,* Verona, Bertani, 1986, p. 73.

heart of mathematical physics, to the "involutive node of mathematical physics"[(78)] ; and the conceptual and ontological depth of this discipline is at the heart of his latest work, which aims to gain a better understanding of the "dialectical structure that generates both abstract mathematical realities and conditions of existence for the universe of phenomena".[(79)] The task of the philosophy of physico-mathematical sciences is therefore to enter into this fact from within in order to deal adequately with "the reasons for the application of Mathematics to the physical universe" and to identify "this uncreated germ which contains within it both the elements of a logical deduction and of an ontological genesis of sensible becoming".[(80)] Lautman continues his "effort of thought" dedicated to taking note of this cognitive situation, to considering the "structures" of mathematics rich in cognitive content and not just as "weapons", as his Bourbakist contemporaries put it[(81)] ; and already in these minor writings, he has endeavoured to understand the epistemic depth of the fruitful collaboration and profound interpenetration between two contents, autonomous but intersecting, between two worlds, the world of mathematics and the world of physics, which make it possible to constantly "remake il 'Timée'" in the words of Jean Petitot[(82)] . In this way, Lautman, like Bachelard, offers us tools capable of going beyond purely linguistic and purist conceptions of mathematics, of grasping beyond its formal and procedural rationality its substantive rationality due to the intrinsic capacity to grasp the structure of reality itself through the objectivity of the increasingly abstract and general forms that it continually generates ; It is for this reason that, from the outset, he has criticised positions that regard it as an auxiliary to physics and, consequently, the limiting visions of an experimental and expressionist imprint. Thus, in his last short but intense work, *Symétrie et dissymétrie en mathématiques et en physique* he insisted on the discontinuous qualitative difference between the mathematical physics of the early twentieth century and earlier physics; in it, for example, algebraic equations

(78) C. Alunni, *Spectres de Bachelard*, op. cit. p. 173.

(79) A. Lautman, *Symétrie et dissymétrie en mathématiques et en physique*, pp. 267-.

(80) *Ibid*, pp. 276-277.

(81) Grothendieck himself, among others, made a criticism of this kind, cf. F. Patras, *La pensée mathématique contemporaine,* op. cit. In this, once again, Lautman was almost a prophet in criticising the underestimation of the problem of the close relationship with physics by the group of Bourbakists; similar criticisms are also present in Simone Weil's contemporary writings on science, already mentioned.

(82) J. Petitot, "Refaire le 'Timée'...", cit.

bring out the qualitative complexity of phenomena by producing autonomous conceptual organisations, as Gaston Bachelard would later say in the last pages of *Rationalisme appliqué*, where he speaks of "topological physics which is not merely a doctrine of quantity.. science [with] the power to know qualities ... and the most numerous nuances".[83]

But there is another dimension, one that is considered essential for philosophical reflection itself because it is seen as structural to scientific knowledge in general, the properly *spiritual* dimension in the broad sense, as is already apparent from these brief writings, which thus enrich the young and often "ungrateful land of the philosophy of science", as his friend Jean Cavaillès called it[84] , with other paths that have not yet been explored[85] ; For Lautman, the philosophy of science has the privilege of always having to do with "methods that give man access to reality" and therefore of "linking the discovery of truth in science to the spiritual progress of a conscience in search of a reality to be known". His strategy of thought is therefore aimed at giving this spiritual dimension a philosophical value of its own, with a contextual critique of all those who had "a purely tautological conception of mathematics" and who thus contributed "through its formalism to rejecting philosophy in favour of the exclusive worship of irrational attitudes".[86] But the "effort of thought" that is being put in place aims to avoid such risks, and must always find from within the necessary resources not to fall into a philosophy of resignation that renounces its primary "heroic" tasks, strengthened by the critical confrontation with the cognitive and spiritual values inherent in the sciences; and not a secondary task of the philosophy of science must be to bring them out, provided that it adequately addresses the "links between thought and reality". It is in this way that the foundations of a genuine 'thought

[83] G. Bachelard, *Le rationalisme appliqué*, op. cit. p. 209.

[84] J. Cavaillès, "Lettre à Albert Lautman", 17 May 1938, in H. Benis-Sinaceur, "Lettres inédites de Jean Cavaillès à Albert Lautman", *Revue d'histoire des sciences*, cit, p. 122.

[85] This 'spiritual' aspect of science had been highlighted in various ways in the context of the French historical-epistemological culture of the early twentieth century by his teacher Léon Brunschvicg and, especially later, by Gaston Bachelard. But the term 'spirit' in the epistemological field designated the basic structure, the true cognitive and ontological 'essence' of science, whose 'revolutions' are also 'revolutions of reason' in short, capable of changing the images of reality and of man himself.

[86] A. Lautman, "Congrès International de Philosophie des Sciences", op. cit. p. 64.

of relation' are built up, and by cultivating these connections we move away from the normative and unilateral positions that every philosophical path, and in particular the philosophy of science, has the task of identifying and avoiding. And in a minor piece of writing that can be considered his spiritual testament, which he leaves us as his legacy, Lautman drew up the programmatic lines of an uncommon path to follow:

> By wishing to eliminate the links between thought and reality, and by refusing to give science the value of a spiritual experience, we run the risk of having only a shadow of science, and of rejecting the mind in search of reality towards violent attitudes in which reason has no part. This is a resignation that the philosophy of science must not accept.[87]

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[87] A. Lautman, "Mathématiques et réalité", cit. p. 50; and this turns out to be one of several attractions, emphasised by many parties, that the research path Lautman set out on produced substantially in just a few years, foreshadowing those other "violent and irrational attitudes" that were to strike the world and cut his life short.